2^{nd} Assignment PH-104 Part B

Part B.a

The satellite stays in his orbit if there is a balance of forces. We assume that $y \in (0,1)$ and $M_P := M_S$

$$F_{cf} = F_{G,M_S} - F_{G,M_P} \textit{ with centrifugal force } F_{cf} = m\omega^2 yR \tag{1}$$

Part B.b

$$\begin{aligned}
F_{cf} &= F_{G,M_S} - F_{G,M_P} &(2)\\
\Leftrightarrow 0 &= F_{cf} + F_{G,M_P} - F_{G,M_S} &(3)\\
\Leftrightarrow 0 &= G^{-1}\omega^2 yR + \tfrac{M_P}{(1-y)^2R^2} - \tfrac{M_S}{y^2R^2} &(4)\\
\Leftrightarrow 0 &= G^{-1}\tfrac{4\pi^2}{T^2}yR + \tfrac{M_P}{(1-y)^2R^2} - \tfrac{M_S}{y^2R^2} &(5)\\
\Leftrightarrow 0 &\underbrace{=}_{\frac{T^2}{R^3}=\frac{4\pi^2}{G(M_S+M_P)}} y(M_S+M_P) + \tfrac{M_P}{(1-y)^2} - \tfrac{M_S}{y^2} &(6)\\
\Leftrightarrow 0 &\underbrace{=}_{\frac{M_P}{M_S}=\eta} (M_S+\eta M_S)y - \tfrac{M_S}{y^2} + \tfrac{\eta M_S}{(1-y)^2} &(7)\\
\Leftrightarrow 0 &= (1+\eta)y - \tfrac{1}{y^2} + \tfrac{\eta}{(1-y)^2} &(8)\\
\Rightarrow f(y) &:= (1+\eta)y - \tfrac{1}{y^2} + \tfrac{\eta}{(1-y)^2} &(9)
\end{aligned}$$

In these transformations we assumed that our parameters are non zero.

Part B.c

We consider y is slightly larger than 0:

$$\lim_{y\to 0+} f(y) = \lim_{y\to 0+} \underbrace{(1+\eta)y}_{\to 0} \underbrace{-\frac{1}{y^2}}_{\to -\infty} \underbrace{+\frac{\eta}{(1-y)^2}}_{\to \eta} < 0 \tag{10}$$

We consider y is slightly smaller than 1:

$$\lim_{y\to 1-} f(y) = \lim_{y\to 1-} \underbrace{(1+\eta)y}_{\to(1+\eta)} \underbrace{-\frac{1}{y^2}}_{\to -1} \underbrace{+\frac{\eta}{(1-y)^2}}_{\to \infty} > 0 \tag{11}$$

Since function f is a continuous function for $y \in [0+\epsilon, 1-\epsilon]$ and $\epsilon > 0$ we can use the "Intermediate value theorem": $\exists \xi$ st. $f(\xi) = 0$ with $\xi \in [0+\epsilon, 1-\epsilon]$.

Part B.d

We set now $\eta = 0.01$:

$y \in (0, 1)$

The solution for $f(y) = 0$ is $y = 0.85763$

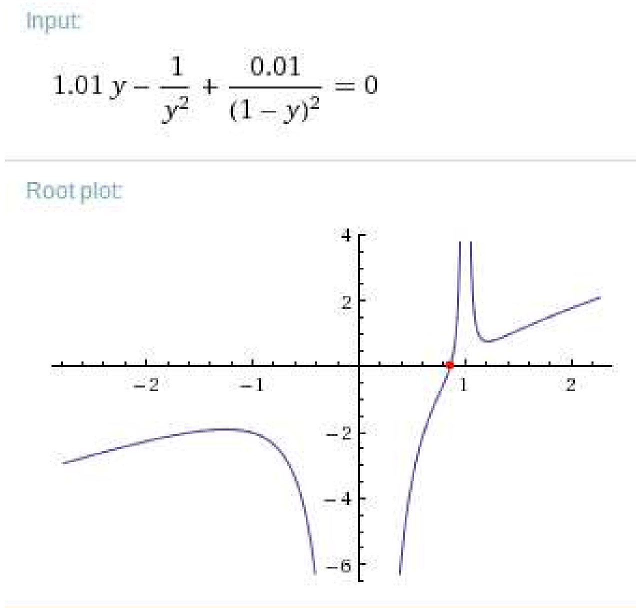

Figure 1: plot of f(y) with $\eta = 0.01$ $y \in (0, 1)$

$y > 1$

For $y \in (1, \infty)$ we need to derive a new function.

$$F_{cf} = F_{G,M_S} + F_{G,M_P} \tag{12}$$

$$\Leftrightarrow 0 = F_{cf} - F_{G,M_P} - F_{G,M_S} \tag{13}$$

$$\Leftrightarrow 0 = G^{-1}\omega^2 yR - \frac{M_P}{(y-1)^2R^2} - \frac{M_S}{y^2R^2} \tag{14}$$

$$\Leftrightarrow 0 = G^{-1}\frac{4\pi^2}{T^2}yR - \frac{M_P}{(y-1)^2R^2} - \frac{M_S}{y^2R^2} \tag{15}$$

$$\Leftrightarrow 0 \underbrace{=}_{\frac{T^2}{R^3}=\frac{4\pi^2}{G(M_S+M_P)}} y(M_S + M_P) - \frac{M_P}{(y-1)^2} - \frac{M_S}{y^2} \tag{16}$$

$$\Leftrightarrow 0 \underbrace{=}_{\frac{M_P}{M_S}=\eta} (M_S + \eta M_S)y - \frac{M_S}{y^2} - \frac{\eta M_S}{(y-1)^2} \tag{17}$$

$$\Leftrightarrow 0 = (1+\eta)y - \frac{1}{y^2} - \frac{\eta}{(y-1)^2} \tag{18}$$

$$\Rightarrow f(y) := (1+\eta)y - \frac{1}{y^2} - \frac{\eta}{(y-1)^2} \tag{19}$$

In these transformations we assumed that our parameters are non zero.
The solution for our new $f(y) = 0$ is $y = 1.15491$

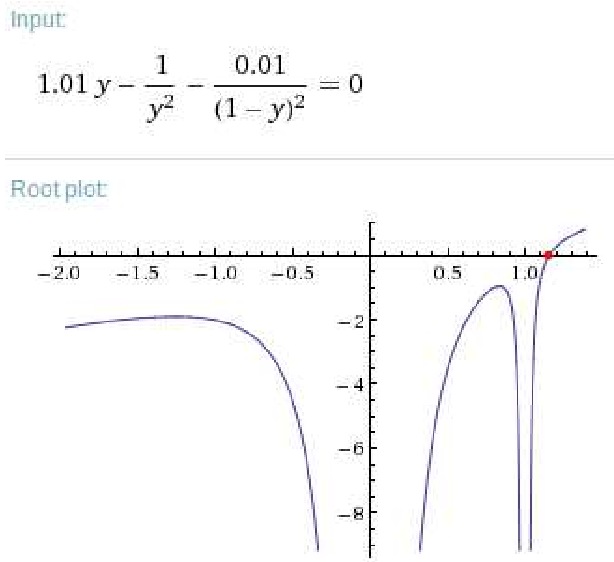

Figure 2: plot of f(y) with $\eta = 0.01$ $y > 1$

$y < 0$

For $y \in (0, -\infty)$ we need to derive a new function.

$$
\begin{aligned}
F_{cf} &= F_{G,M_S} + F_{G,M_P} && (20)\\
\Leftrightarrow 0 &= F_{cf} - F_{G,M_P} - F_{G,M_S} && (21)\\
\Leftrightarrow 0 &= G^{-1}\omega^2 yR - \tfrac{M_P}{(y+1)^2R^2} - \tfrac{M_S}{y^2R^2} && (22)\\
\Leftrightarrow 0 &= G^{-1}\tfrac{4\pi^2}{T^2} yR - \tfrac{M_P}{(y+1)^2R^2} - \tfrac{M_S}{y^2R^2} && (23)\\
\Leftrightarrow 0 &\underbrace{=}_{\frac{T^2}{R^3}=\frac{4\pi^2}{G(M_S+M_P)}} y(M_S + M_P) - \tfrac{M_P}{(y+1)^2} - \tfrac{M_S}{y^2} && (24)\\
\Leftrightarrow 0 &\underbrace{=}_{\frac{M_P}{M_S}=\eta} (M_S + \eta M_S)y - \tfrac{M_S}{y^2} - \tfrac{\eta M_S}{(y+1)^2} && (25)\\
\Leftrightarrow 0 &= (1+\eta)y - \tfrac{1}{y^2} - \tfrac{\eta}{(y+1)^2} && (26)\\
\Rightarrow f(y) &:= (1+\eta)y - \tfrac{1}{y^2} - \tfrac{\eta}{(y+1)^2} && (27)
\end{aligned}
$$

In these transformations we assumed that our parameters are non zero.
The solution for our new $f(y) = 0$ is $y = 0.997517$

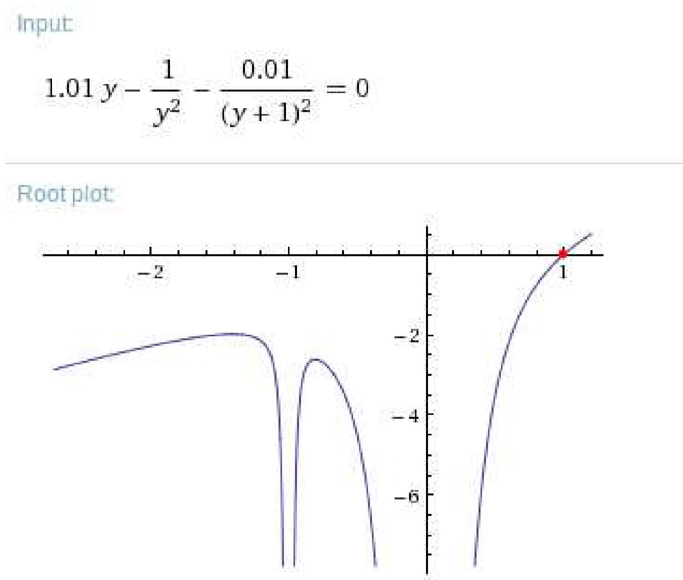

Figure 3: plot of f(y) with $\eta = 0.01$ $y < 0$