1. Die Herleitung der Planck Einheiten anhand der Energieimpulsformel

Die Planck-Länge:

$$Ep = m^2c^3 = F_p\hbar$$

$$F_G = G\frac{m^2}{l_p^2}$$

$$m^2c^3 = G\frac{m^2}{l_p^2}\hbar$$

$$l_p = \sqrt{\frac{G\hbar}{c^3}}$$

Die Planck-Zeit:

$$l_p = \sqrt{\frac{G\hbar}{c^3}}$$

$$t = \frac{l_p}{c} = \frac{\sqrt{\frac{G\hbar}{c^3}}}{c} = \sqrt{\frac{G\hbar}{c^5}}$$

Die Planck-Masse:

$$Ep = m^2 c^3 = F_p \hbar$$

$$l_p = \sqrt{\frac{G\hbar}{c^3}}$$

$$\frac{m}{E} mc^3 = \frac{F_p}{E} \hbar$$

$$\frac{1}{c^2} mc^3 = \frac{1}{l_p} \hbar$$

$$m = \frac{\hbar}{l_p c} = \sqrt{\frac{\hbar c}{G}}$$

Die Planck-Ladung:

$$Ep = m^2c^3 = F_p\hbar$$

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l_p^2}$$

$$l_p = \sqrt{\frac{G\hbar}{c^3}}$$

$$m = \sqrt{\frac{\hbar c}{G}}$$

$$m^2c^3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l_p^2} = \frac{\hbar c}{G}c^3 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\frac{G\hbar}{c^3}}\hbar \quad \Rightarrow \qquad q = \sqrt{4\pi\varepsilon_0\hbar c}$$