1. $$m\frac{d^2x}{dt^2} = -kx$$

$$m\frac{d^2x}{dt^2}\frac{dx}{dt} = -kx\frac{dx}{dt}$$

$$m\int_{t_0}^{t}\frac{d^2x}{dt^2}\frac{dx}{dt}dt = -k\int_{t_0}^{t}x\frac{dx}{dt}dt$$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 - \frac{m}{2}\left(\frac{dx\left(t_0\right)}{dt}\right)^2 = -\frac{k}{2}x^2 + \frac{k}{2}x^2\left(t_0\right)$$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = -\frac{k}{2}x^2 + C$$

Homogene Lösung

$$\frac{1}{x}\frac{dx}{dt} = \sqrt{\frac{-k}{m}}$$

$$\int_{t_0}^{t}\frac{1}{x}\frac{dx}{dt}dt = \int_{t_0}^{t}\sqrt{\frac{-k}{m}}dt$$

$$\ln(x(t)) - \ln\left(x\left(t_0\right)\right) = \sqrt{\frac{-k}{m}}t - \sqrt{\frac{-k}{m}}t_0$$

$$\ln\left(\frac{x(t)}{x\left(t_0\right)}\right) = \sqrt{\frac{-k}{m}}t - \sqrt{\frac{-k}{m}}t_0$$

$$\frac{x(t)}{x\left(t_0\right)} = e^{i\sqrt{\frac{k}{m}}(t-t_0)}$$

$$x(t) = x\left(t_0\right)e^{i\sqrt{\frac{k}{m}}(t-t_0)}$$

Partikuläre Lösung

$$\frac{m}{2}0^2 + \frac{k}{2}x_p^2 = C$$

$$x_p = \sqrt{\frac{2C}{k}}$$

Vollständige Lösung

$$x(t) = x\left(t_0\right)\left(\cos\left(\sqrt{\frac{k}{m}}\left(t - t_0\right)\right) + i\sin\left(\sqrt{\frac{k}{m}}\left(t - t_0\right)\right) + \sqrt{\frac{2C}{k}}\right.$$