$$\langle n' l' m_l' m_s' \mid L_y S_y \mid n l m_l m_s \rangle$$

$$= -\frac{\hbar^2}{4} \sqrt{l(l+1) - m_l(m_l+1)} \Big(\sqrt{s(s+1) - m_s(m_s+1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l+1) m_l'} \delta_{(m_s+1) m_s'}$$

$$- \sqrt{s(s+1) - m_s(m_s-1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l+1) m_l'} \delta_{(m_s-1) m_s'} \Big)$$

$$+ \frac{\hbar^2}{4} \sqrt{l(l+1) - m_l(m_l-1)} \Big(\sqrt{s(s+1) - m_s(m_s+1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l-1) m_l'} \delta_{(m_s+1) m_s'}$$

$$- \sqrt{s(s+1) - m_s(m_s-1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l-1) m_l'} \delta_{(m_s-1) m_s'} \Big)$$

$$\langle n' l' m_l' m_s' \mid L_x S_x \mid n l m_l m_s \rangle + \langle n' l' m_l' m_s' \mid L_y S_y \mid n l m_l m_s \rangle$$

$$= \frac{\hbar^2}{2} \sqrt{l(l+1) - m_l(m_l+1)} \sqrt{s(s+1) - m_s(m_s-1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l+1) m_l'} \delta_{(m_s-1) m_s'}$$

$$+ \frac{\hbar^2}{2} \sqrt{l(l+1) - m_l(m_l-1)} \sqrt{s(s+1) - m_s(m_s+1)}\, \delta_{nn'} \delta_{ll'} \delta_{(m_l-1) m_l'} \delta_{(m_s+1) m_s'}$$