$$\chi^+ \chi =$$

$$-\frac{1}{4m^2c^2}\left(1-\frac{T+e\varphi}{2mc^2}\right)\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(1-\frac{T+e\varphi}{2mc^2}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$$=\left(-\frac{1}{4m^2c^2}+\underbrace{\frac{T+e\varphi}{8m^3c^4}}_{\to 0}\right)\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(1-\frac{T+e\varphi}{2mc^2}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$$\approx -\frac{1}{4m^2c^2}\,\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(1-\frac{T+e\varphi}{2mc^2}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$$=\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(-\frac{1}{4m^2c^2}+\underbrace{\frac{T+e\varphi}{8m^3c^4}}_{\to 0}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$$\approx \hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(-\frac{1}{4m^2c^2}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$$=\hat{\psi}^+\left(-\frac{1}{4m^2c^2}(\hat{\vec{p}}\cdot\vec{\sigma})^2\right)\hat{\psi}$$

$$\hat{\psi}^+\hat{\psi}+\chi^+\chi$$

$$=\hat{\psi}\left(1-\frac{1}{4m^2c^2}(\hat{\vec{p}}\cdot\vec{\sigma})^2\right)\hat{\psi}$$

?

$$\left((\hat{\vec{p}}\cdot\vec{\sigma})\hat{\psi}\right)^+$$
$$=\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})^+$$
$$=\hat{\psi}^+\sum_i(\sigma_i\cdot\hat{p}_i)^+$$
$$=\hat{\psi}^+\sum_i\hat{p}_i^*\,\sigma_i^+$$
$$=-\hat{\psi}^+(\hat{\vec{p}}\cdot\vec{\sigma})$$