$$\chi^+ = \frac{1}{2mc}\left(1 - \frac{T + e\varphi}{2mc^2} + O\left(\frac{v^4}{c^4}\right)\right)\left((\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}\right)^+$$

$$= \frac{1}{2mc}\left(1 - \frac{T + e\varphi}{2mc^2} + O\left(\frac{v^4}{c^4}\right)\right)\left(-\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\right)$$

$\chi^+\chi$

$$= -\frac{1}{4m^2c^2}\left(1 - \frac{T + e\varphi}{2mc^2}\right)\hat{\psi}^+(\vec{\sigma}\cdot\hat{\vec{p}})\left(1 - \frac{T + e\varphi}{2mc^2}\right)(\vec{\sigma}\cdot\hat{\vec{p}})\,\hat{\psi}$$

$(\vec{\sigma}\cdot\hat{\vec{p}}) = (\hat{\vec{p}}\cdot\vec{\sigma})$, $T = E - mc^2$

$\varphi = \varphi(\vec{r})$

$$
\begin{aligned}
|\chi\rangle &= \frac{1}{2m_\mathrm{e}\,c}\left(1 + \frac{T + e\,\varphi}{2m_\mathrm{e}\,c^2}\right)^{-1} (\boldsymbol{p}\cdot\boldsymbol{\sigma})|\widehat{\psi}\rangle \\
&= \frac{1}{2m_\mathrm{e}\,c}\left[1 - \frac{T + e\,\varphi}{2m_\mathrm{e}\,c^2} + 0\left(\frac{v^4}{c^4}\right)\right] (\boldsymbol{p}\cdot\boldsymbol{\sigma})|\widehat{\psi}\rangle\,. \qquad (5.247)
\end{aligned}
$$

$$\langle\psi|\psi\rangle \stackrel{!}{=} \langle\eta|\eta\rangle = \langle\widehat{\psi}|\alpha^{-2}|\widehat{\psi}\rangle = \langle\widehat{\psi}|\widehat{\psi}\rangle + \langle\chi|\chi\rangle$$

$$\stackrel{(5.247)}{=} \langle\widehat{\psi}|\left[1 + \frac{1}{4m_e c^2}(\boldsymbol{p}\cdot\boldsymbol{\sigma})^2 + \mathcal{O}\left(\frac{v^4}{c^4}\right)\right]|\widehat{\psi}\rangle$$