# CRYOGENIC ENGINEERING

**Prof. Milind D. Atrey**

Department of Mechanical Engineering,
IIT Bombay

**Lecture No - 34**

# Earlier Lecture

- Cryogenic vessels use insulation to minimize all modes of heat transfer.
- Apparent thermal conductivity ($\mathbf{k_A}$) is calculated based on all possible modes of heat transfer.
- Expanded foam is a low density, cellular structure. A gas filled powder or a fibrous insulation reduces the gas convection due to the small size of voids.
- Radiation heat transfer is reduced by using radiation shields.

# Outline of the Lecture

## Topic : Cryogenic Insulation (contd)

- Vacuum
- Evacuated Powders
- Opacified Powders
- Tutorial

# Types of Insulation

- Expanded Foam – Mass
- Gas Filled Powders & Fibrous Materials – Mass
- Vacuum alone – Vacuum
- Evacuated Powders – Mass + Vacuum
- Opacified Powders – Mass + Vacuum + Reflective
- Multilayer Insulation – Vacuum + Reflective

# Introduction

- As seen earlier, the different modes of heat transfer are Conduction, Convection and Radiation.
- If the physical matter between the hot and the cold surfaces is removed, that is, by maintaining a perfect vacuum, Conduction and Convection are eliminated.
- However, Radiation heat transfer does not require any medium and in such cases, it is the only mode of heat transfer.

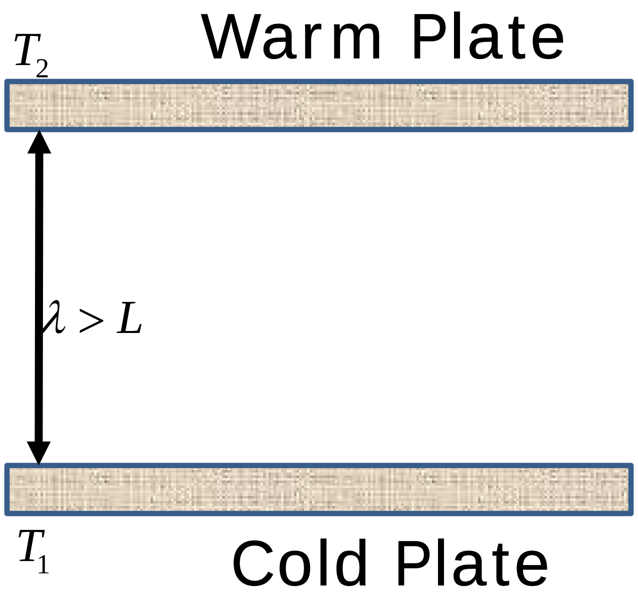
# Vacuum

- It is important to note that even in vacuum, there is some residual gas.
- These gas molecules contribute to the heat transfer by gaseous conduction.
- As the vacuum improves, this gas conduction decreases.
- In an ordinary conduction, a linear temperature gradient is built up. The molecules exchange heat with each other and as well as with the surfaces.

# Vacuum

- But in vacuum, the mean free path (**λ**) of the molecules is more than the distance between the surfaces; the molecules rarely collide with each other.
- The energy is exchanged only between the surface and the colliding molecules.
- This type of heat transfer is called as free molecular conduction or residual gas conduction.
- This exists only at very low pressures or at very good vacuum.

# Vacuum

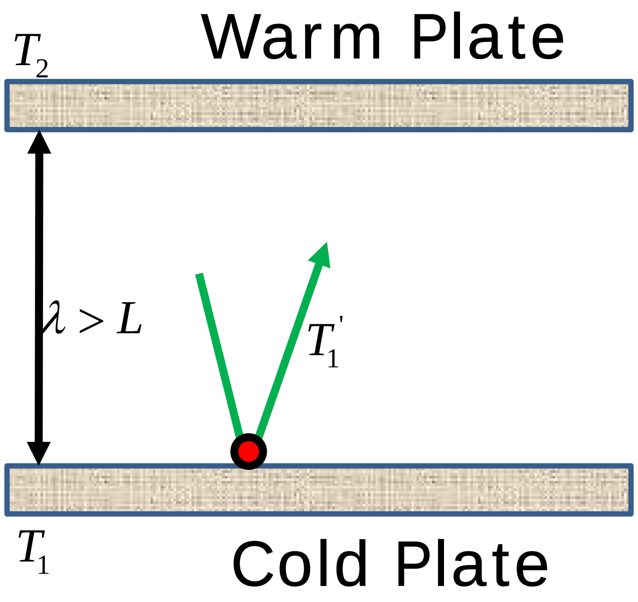



- For the sake of understanding, consider two plates with temperatures **$T_1$** and **$T_2$**, (**$T_2$ > $T_1$**) as shown.

- The gas pressure is very low in order to ensure that the mean free path (**λ**) of the molecules is greater than **L**.

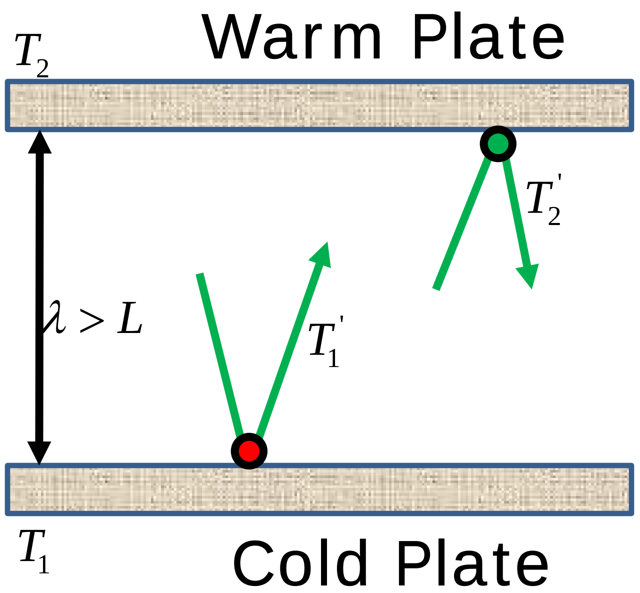
- In such situations, the gas molecules collide only with the surfaces and exchange energy.

# Vacuum




- Consider a molecule colliding with bottom plate and leaving towards upper plate.
- The gas molecule collides with this surface at $\mathbf{T_1}$ and it transfers some energy to the surface.
- It leaves the cold surface with a kinetic energy corresponding to a temperature $\mathbf{T'_1}$, higher than $\mathbf{T_1}$.

# Vacuum

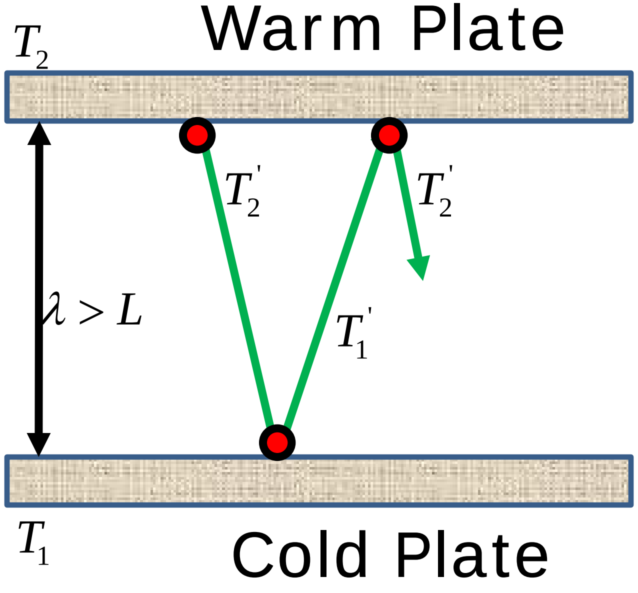



- Again, consider a molecule colliding with upper plate and leaving towards bottom plate.
- This gas molecule collides with surface at $\mathbf{T_2}$ and leaves at a temperature $\mathbf{T'_2}$, lower than $\mathbf{T_2}$.
- It is clear that, in both these impacts, thermal equilibrium is not attained. This process is repeated and contributes to free molecular conduction.

# Vacuum

- In order to measure the degree of thermal equilibrium between the molecule and the surface, we define Accommodation Coefficient (**a**).
- It is a ratio of actual energy transfer to the maximum possible energy transfer.
- Mathematically, $$a = \frac{Actual\ \ Heat\ Transfer}{Max\ \ Heat\ \ Transfer}$$
- Its value depends on the gas – surface interaction and the temperature of the surface.

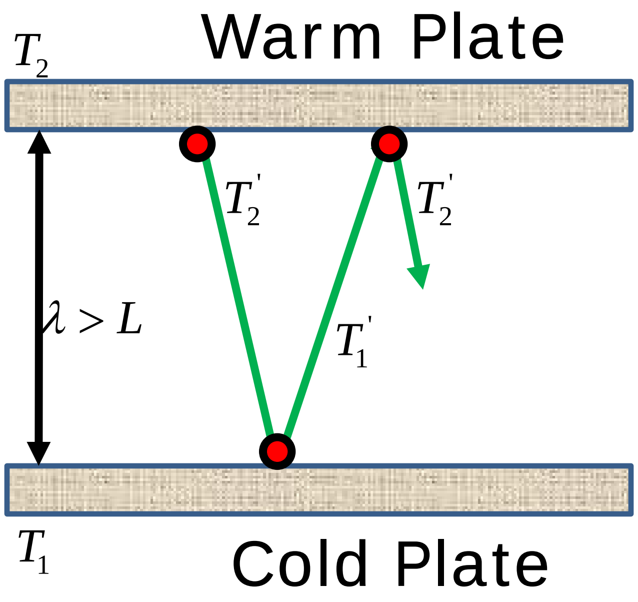
# Vacuum




- From the figure, for the cold surface, the actual temperature change is **($T'_2$ – $T'_1$)**.
- But, the maximum possible temperature change is **($T'_2$ – $T_1$)**.
- By definition, the accommodation coefficient for cold plate is

$$a_1 = \frac{T_2^{'} - T_1^{'}}{T_2^{'} - T_1}$$

# Vacuum




- Similarly, for the hot surface, the actual temperature change is **($T'_2$ – $T'_1$)**.
- But, the maximum possible temperature change is **($T_2$ – $T'_1$)**.
- Therefore, the accommodation coefficient for the hot surface is given by

$$a_2 = \frac{T_2' - T_1'}{T_2 - T_1'}$$

# Vacuum

- From the earlier slides, the accommodation coefficients are

$$a_1 = \frac{T_2' - T_1'}{T_2' - T_1} \qquad a_2 = \frac{T_2' - T_1'}{T_2 - T_1'}$$

- Rearranging the above equations, we have

$$T_1 = T_2' - \frac{\boxed{T_2' - T_1'}}{a_1} \qquad T_2 = \frac{\boxed{T_2' - T_1'}}{a_2} + T_1'$$

$$T_2 - T_1 = \left(T_2' - T_1'\right)\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$

# Vacuum

$$T_2 - T_1 = \left(T_2' - T_1'\right)\boxed{\left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)}$$

- Similar to an emissivity factor, we define a term accommodation factor $\mathbf{F_a}$, which is given by

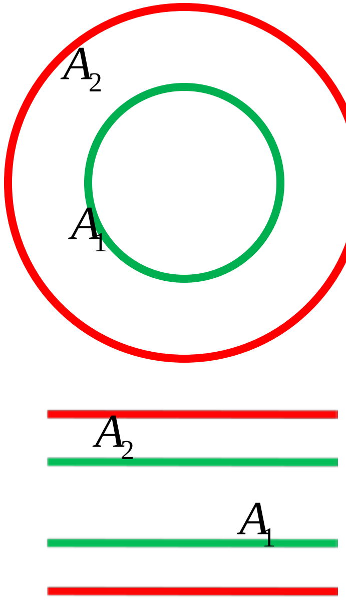
$$\frac{1}{F_a} = \left(\frac{1}{a_1} + \frac{1}{a_2} - 1\right)$$

$$T_2 - T_1 = \left(T_2' - T_1'\right)\frac{1}{F_a}$$

$$F_a = \frac{T_2' - T_1'}{T_2 - T_1}$$

# Vacuum

- The approximate accommodation coefficients for concentric sphere and concentric cylinder geometries are as tabulated below.




| Temp (K) | He | $H_2$ | Ne | Air |
|---|---|---|---|---|
| 300 | 0.29 | 0.29 | 0.66 | 0.8-0.9 |
| 78 | 0.42 | 0.53 | 0.83 | 1.0 |
| 20 | 0.59 | 0.97 | 1.0 | 1.0 |

- The subscript **1** denotes the enclosed surface and subscript **2** denotes the enclosure.

# Vacuum

| Temp (K) | He | $H_2$ | Ne | Air |
|---|---|---|---|---|
| 300 | 0.29 | 0.29 | 0.66 | 0.8-0.9 |
| 78 | 0.42 | 0.53 | 0.83 | 1.0 |
| 20 | 0.59 | 0.97 | 1.0 | 1.0 |

- At a given temperature, the accommodation coefficient increases with the increase in the molecular weight of the gas.

- For a given gas, the accommodation coefficient increases with the decrease in the temperature, due to better heat transfer at lower temperatures.

# Vacuum

- From the kinetic theory of gases, the total energy of a molecule is the sum of internal energy and kinetic energy.

- Mathematically,

$$e = U + KE$$

$$e = \left(c_v + \frac{R}{2}\right)T$$

$$\Delta e = \left(c_v + \frac{R}{2}\right)\Delta T$$

- where,
  - R – Specific gas constant
  - $c_v$ = **R / (γ-1)** – Specific heat of gas
  - ΔT = **$(T'_2 - T'_1)$** – Change in temperature

# Vacuum

$$\Delta e = \left( c_v + \frac{R}{2} \right) \Delta T$$

- The definition of **$C_v$** and **$F_a$** are as given below.

$$c_v = \frac{R}{\gamma - 1}$$

$$T_2^{'} - T_1^{'} = F_a \left( T_2 - T_1 \right)$$

- Substituting, we have

$$\Delta e = \left( \frac{R}{\gamma - 1} + \frac{R}{2} \right) \left( T_2 - T_1 \right) F_a$$

$$\Delta e = \frac{F_a R}{2} \left( T_2 - T_1 \right) \left( \frac{\gamma + 1}{\gamma - 1} \right)$$

# Vacuum

- The mass flux per unit time is given by

$$\frac{\dot{m}}{A} = \frac{\rho\bar{\upsilon}}{4}$$

- where,
  - ρ – Density, $\bar{\upsilon}$ – Average velocity

- From Kinetic theory, average velocity is $\bar{\upsilon} = \left(\frac{8RT}{\pi}\right)^{0.5}$

- Combining the above, together with equation of state, we have

$$\frac{\dot{m}}{A} = \frac{1}{4}\left(\frac{p}{RT}\right)\left(\frac{8RT}{\pi}\right)^{0.5} \qquad \frac{\dot{m}}{A} = p\left(\frac{1}{2\pi RT}\right)^{0.5}$$

# Vacuum

- The total energy transfer per unit area owing to the molecular conduction is as given below.

$$\frac{\dot{Q}}{A} = \boxed{\frac{\dot{m}}{A}}\boxed{(\Delta e)}$$

$$\boxed{\frac{\dot{m}}{A}} = p\left(\frac{1}{2\pi RT}\right)^{0.5}$$

$$\boxed{\Delta e} = \frac{F_a R}{2}(T_2 - T_1)\left(\frac{\gamma+1}{\gamma-1}\right)$$

$$\frac{\dot{Q}}{A} = p\left(\frac{1}{2\pi RT}\right)^{0.5}\left(\frac{F_a R}{2}(T_2 - T_1)\left(\frac{\gamma+1}{\gamma-1}\right)\right)$$

$$\frac{\dot{Q}}{A} = \left(\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{R}{8\pi T}\right)^{0.5} F_a\right)p(T_2 - T_1)$$

- **T** is the temperature of the pressure gauge measuring the gas pressure.

# Vacuum

$$\frac{\dot{Q}}{A} = \boxed{\left( \left( \frac{\gamma+1}{\gamma-1} \right) \left( \frac{R}{8\pi T} \right)^{0.5} F_a \right)} p(T_2 - T_1)$$

- In the above equation, let us denote the term in the parenthesis by **G**. We have,

$$\dot{Q} = G \ p \ A \ (T_2 - T_1)$$

- **Q** is valid only when the distance (**L**) between the plates is less than the mean free path (**λ**). Mathematically,

$$L < \lambda = \frac{\mu}{p} \left( \frac{\pi R T}{2} \right)^{0.5}$$

# Vacuum

$$\dot{Q} = GpA(T_2 - T_1)$$

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

- From the above two equations, it is clear that the
  - The free molecular regime can be achieved by achieving very good vacuum.
  - The free molecular conduction heat transfer can be made negligible compared to other modes, by lowering the pressure, decreasing $F_a$, decreasing $(\mathbf{T_2} - \mathbf{T_1})$.

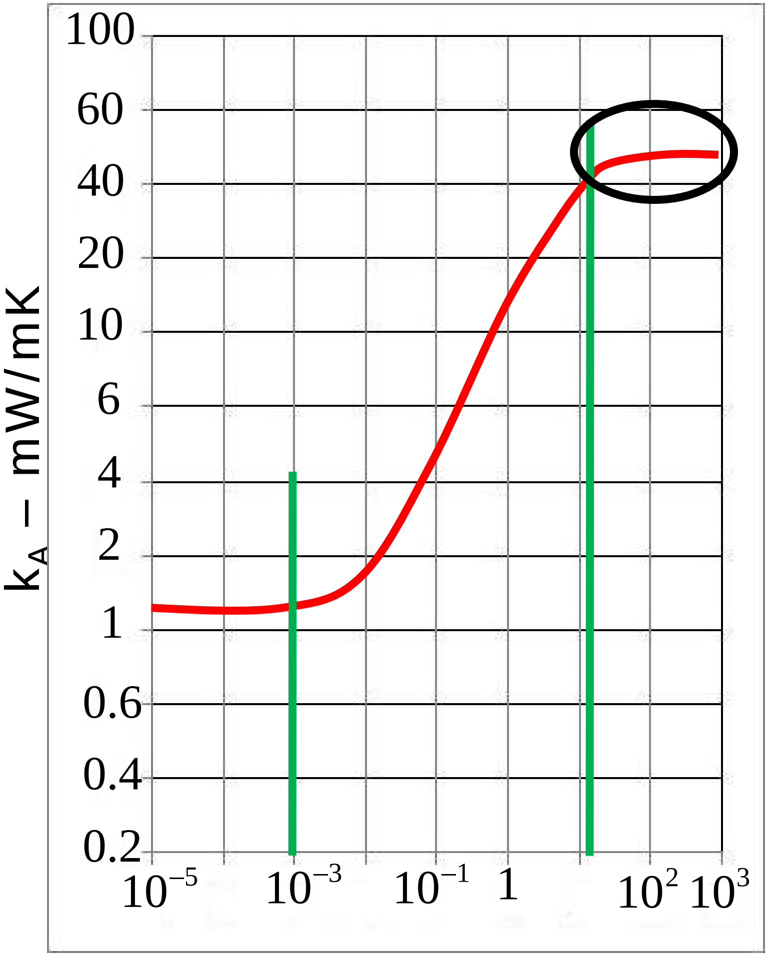
# Evacuated Powder

- Gas conduction is the primary and the dominant mode of heat transfer in a gas filled powder and fibrous insulations.

- One of the obvious ways to reduce this heat transfer is to evacuate the powder and the fibrous insulations.

- Usually, the vacuum that is commonly maintained in these insulations is in the range of $\mathbf{10^{3}}$ to $\mathbf{10^{-5}}$ torr. **1** torr = 1 mm of Hg.

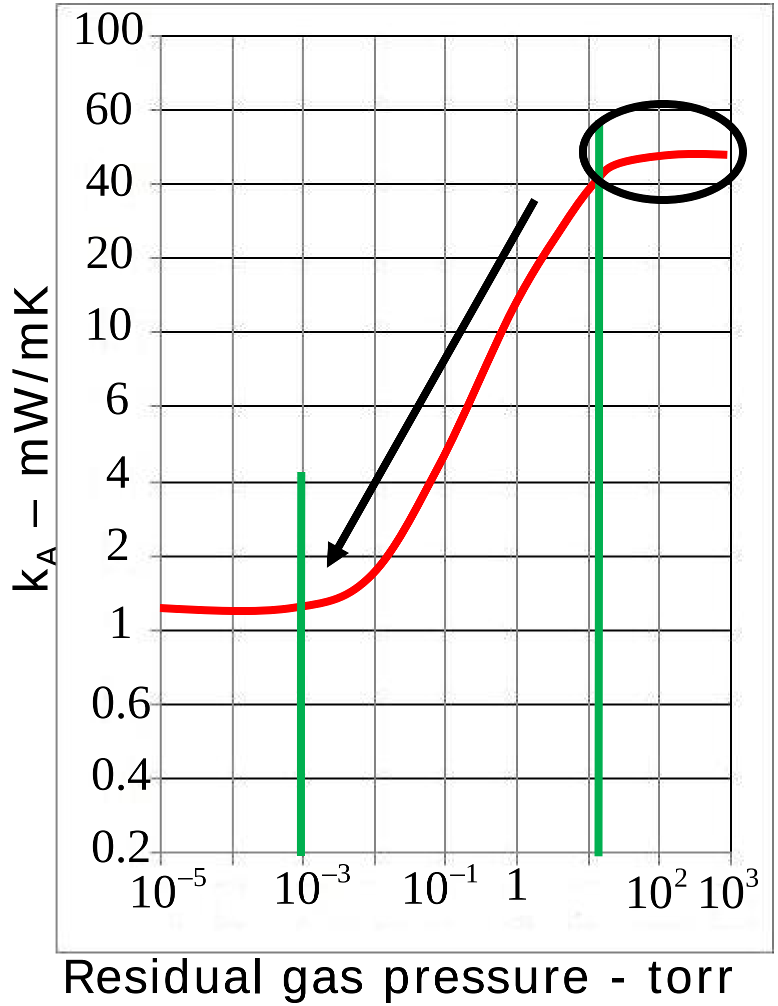
# Evacuated Powder



Residual gas pressure - torr

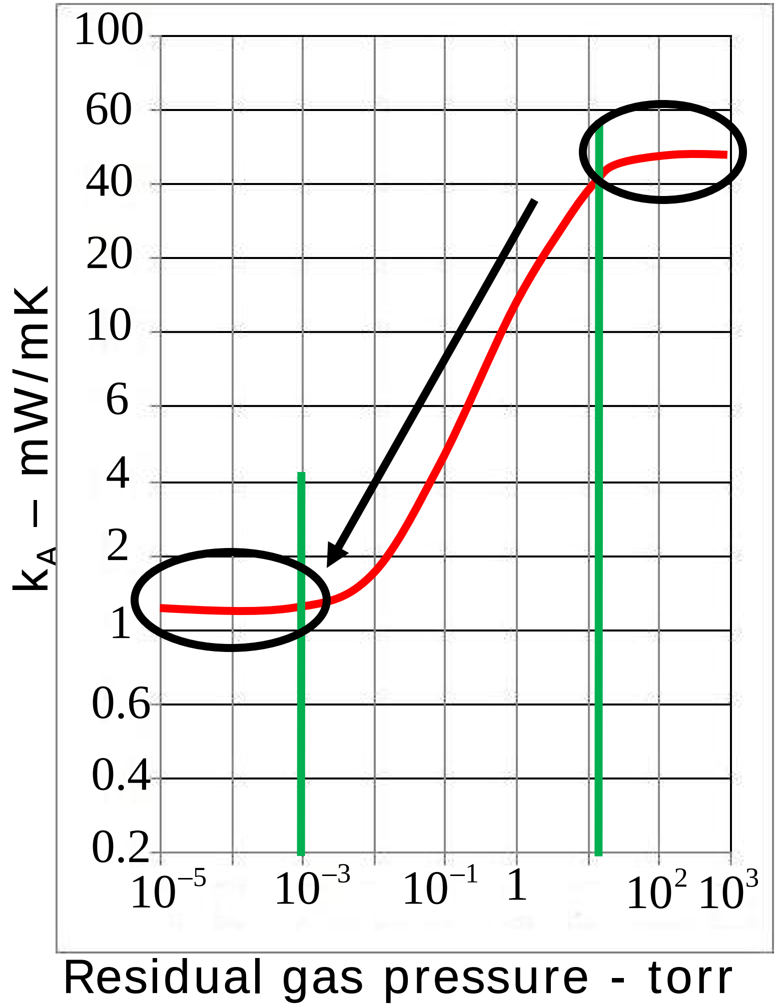
- The adjacent figure shows the variation of $\mathbf{k_A}$ with the residual gas pressure inside an evacuated powder insulation.
- $\mathbf{k_A}$ is independent of residual gas pressures lying between atmospheric and 15 torr.

# Evacuated Powder



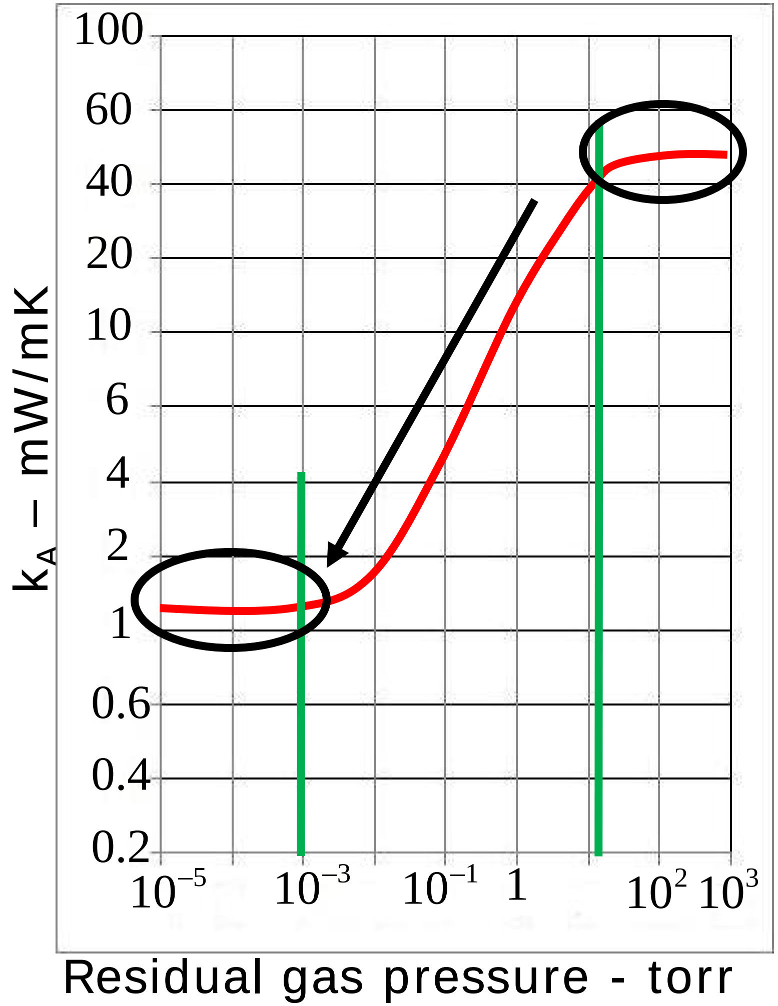

- With the lowering of pressure, 15 torr to $10^{-3}$ torr, $\mathbf{k_A}$ becomes directly proportional to the pressure.
- It varies almost linearly on a logarithmic chart as shown.
- Here, the modes of heat transfer are due to radiation, solid conduction and free molecular conduction (dominant).

# Evacuated Powder




- With the further lowering of pressure, below **$10^{-3}$** torr, the variation of **$k_A$** is almost null.
- The mode of heat transfer is primarily due to solid conduction and radiation.
- Evacuated powders are superior in performance than vacuum alone in **300-77 K**, as the radiation heat transfer is comparatively less.

# Evacuated Powder




- At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.

- Hence, it is more advantageous to use vacuum alone in **77 K** to **4 K**. From Fourier's Law, we have

$$\dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x}$$

# Evacuated Powder

$$\dot{Q} = \frac{k_A A_m (T_h - T_c)}{\Delta x}$$

- where,
  - $k_A$ = Apparent thermal conductivity
  - $T_h - T_c$ = Temperature difference
  - $\Delta x$ = Distance
  - $A_m$ = Mean area of insulation. $\mathbf{A_m}$ for concentric cylinders and concentric spheres is as given below.

$$A_{m,cyl} = \frac{A_2 - A_1}{\ln {A_2}/{A_1}}$$

$$A_{m,sph} = \left(A_1 A_2\right)^{\frac{1}{2}}$$

# Evacuated Powder

- The apparent thermal conductivity and density of few commonly used evacuated powder insulations are as shown.

- The residual gas pressure is less than $10^{-3}$ torr for temperatures between **77 K** to **300 K**.

| Powder | ρ (kg / m³) | k (mW / mK) |
|---|---|---|
| Fine Pertile | 180 | 0.95 |
| Coarse Perlite | 64 | 1.90 |
| Lampblack | 200 | 1.20 |
| Fiberglass | 50 | 1.70 |

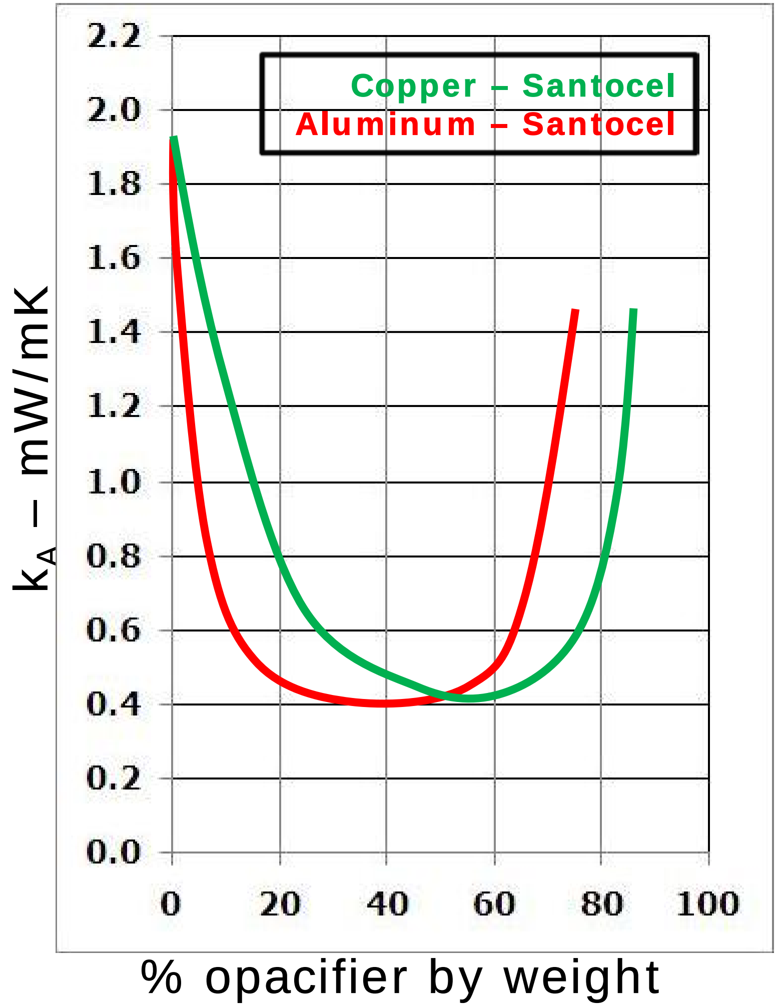
# Opacified Powder Insulation

- Radiation heat transfer still contributes to the heat in leak in **300 K** to **77 K** temperature range in case of evacuated powders.

- In the year 1960, **Riede** and **Wang**, **Hunter** *et. al*. minimized this radiant heat transfer by addition of reflective flakes made of **Al** or **Cu** to the evacuated powder.

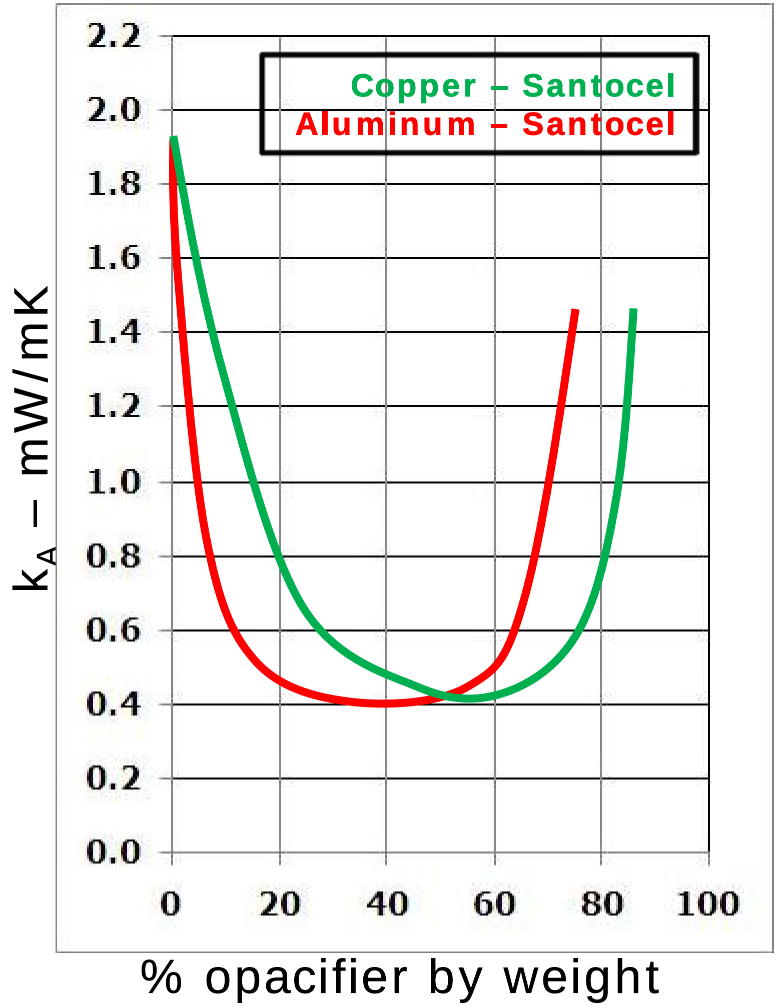
- These flakes act like radiant shields in the tiny heat transfer paths that are formed in the interstices of the evacuated powder.

# Opacified Powder Insulation



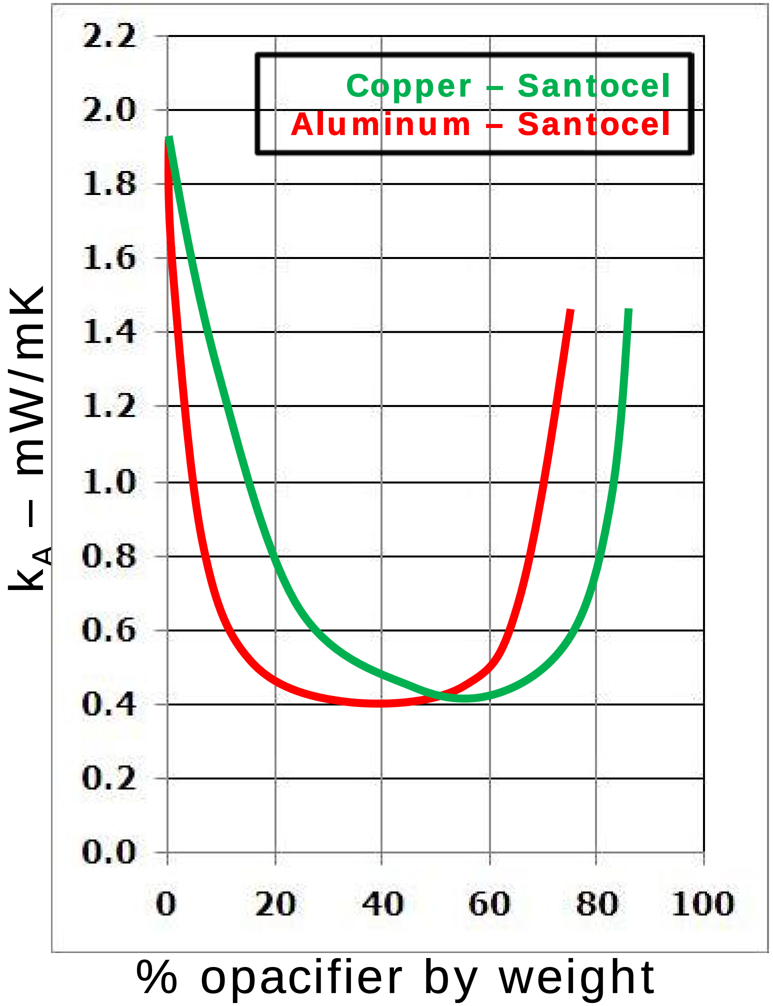

- The figure shows the variation of % opacifier with thermal conductivity for **Cu – santocel** and **Al – santocel**.
- There exists an optimum operating point for each of these insulations.
- It has been observed that, with these additions, $\mathbf{k_A}$ can be reduced by 5 times.

# Opacified Powder Insulation




- **Cu** flakes are more preferred as compared to **Al** flakes.
- The **Al** flakes have large heat of combustion.
- These together with $\mathbf{O_2}$ can lead to accidents when used on **LOX** containers.

# Opacified Powder Insulation




- Another disadvantage of this insulation is that the vibrations tend to pack the flakes together.

- This, not only increases the thermal conductivity but also short circuits the conduction heat transfer.

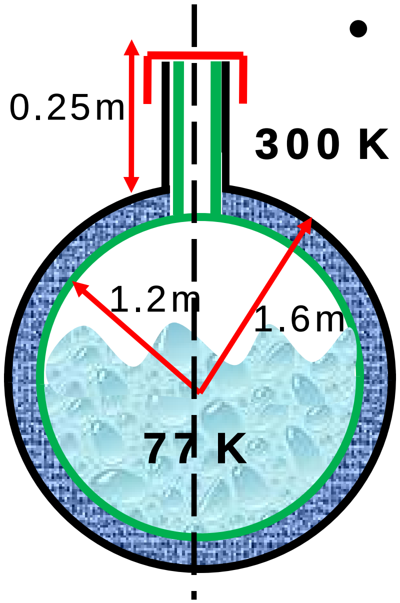
# Opacified Powder Insulation

- The apparent thermal conductivity (mW/mK) and density (kg/m$^3$) of few commonly used opacified powder insulations are as shown.

- The residual gas pressure is less than $10^{-3}$ torr for temperatures between **77 K** to **300 K**.

| Powder | ρ (kg/m$^3$) | k (mW/mK) |
|---|---|---|
| 50/50 Cu – Santocel | 180 | 0.33 |
| 40/60 Al – Santocel | 160 | 0.35 |
| 50/50 Bronze – Santocel | 179 | 0.58 |
| Silica – Carbon | 80 | 0.48 |

# Tutorial




- A spherical **LN2** vessel (**e**=0.8) is as shown. The inner and outer radii are 1.2m and 1.6m respectively. Compare and comment on the heat in leak for the following cases.

  - Perlite (26 mW/mK)
  - Less Vacuum (1.5mPa)
  - Vacuum alone
  - Vacuum + 10 shields ($\mathbf{e_s}$=0.05)
  - Evacuated Fine Perlite (0.95 mW/mK)
  - 50/50 Cu – Santocel (0.33 mW/mK)

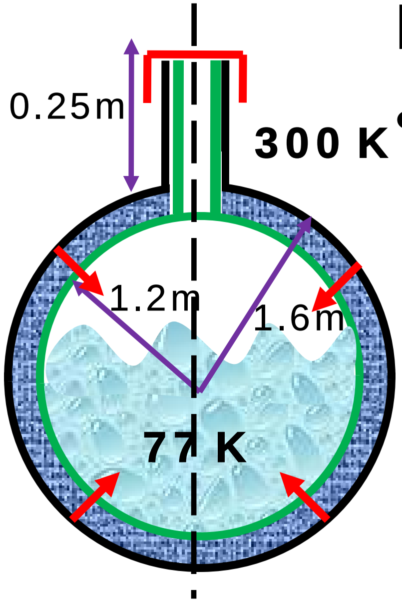
# Tutorial

| Given |
|---|
| Apparatus : Spherical vessel (**e**=0.8) |
| Working Fluid : Liquid Nitrogen |
| Temperature : 77 K (inner), 300 K (outer) |

| Calculate heat in leak | |
|---|---|
| **1** | Perlite (26 mW/mK) |
| **2** | Less Vacuum (1.5mPa) |
| **3** | Vacuum alone |
| **4** | Vacuum + 10 shields |
| **5** | Evacuated Fine Perlite (0.95 mW/mK) |
| **6** | 50/50 Cu – Santocel (0.33 mW/mK) |

- The shape factor between the two containers is assumed to be 1.

# Tutorial




Perlite ($k_A$ = 26mW/m-K)

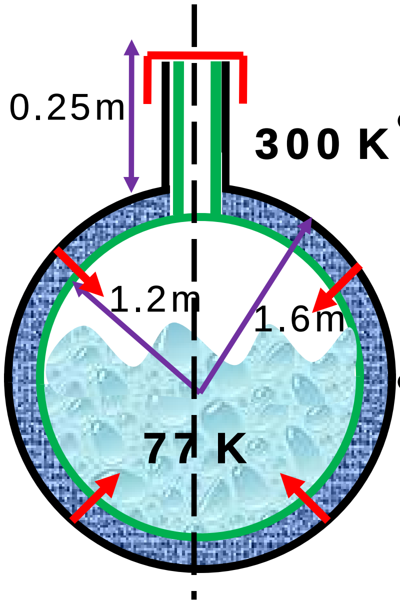
- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $k_A$, $\Delta T$=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = \frac{4\pi\left(26\right)\left(10^{-3}\right)\left(1.6\right)\left(1.2\right)\left(223\right)}{\left(1.6 - 1.2\right)}$$

$$Q = 349.7W$$

# Tutorial



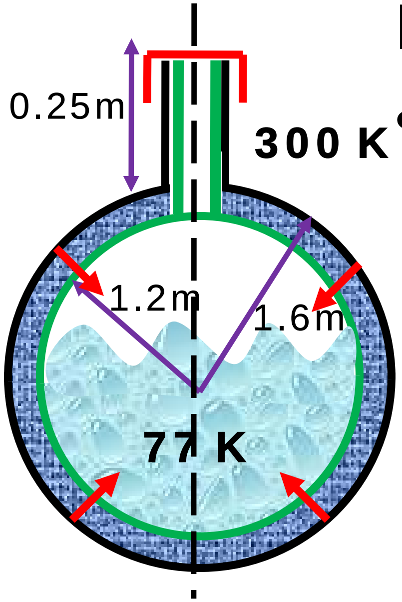

Less Vacuum (1.5mPa)

- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $e_1$=$e_2$=0.8, $T_1$=77 K, $T_2$=300 K.
- The net heat transfer is due to both radiation and residual gas conduction.

$$F_e = \left( \frac{1}{e_1} + \left( \frac{A_1}{A_2} \right) \left( \frac{1}{e_2} - 1 \right) \right)^{-1}$$

$$F_e = \left( \frac{1}{0.8} + \left( \frac{1.2}{1.6} \right)^2 \left( \frac{1}{0.8} - 1 \right) \right)^{-1} = 0.72$$

# Tutorial




Less Vacuum (1.5mPa)

- **Sphere** - $R_1=1.6m$, $R_2=1.2m$, $e_1=e_2=0.8$, $T_1=77$ K, $T_2=300$ K.

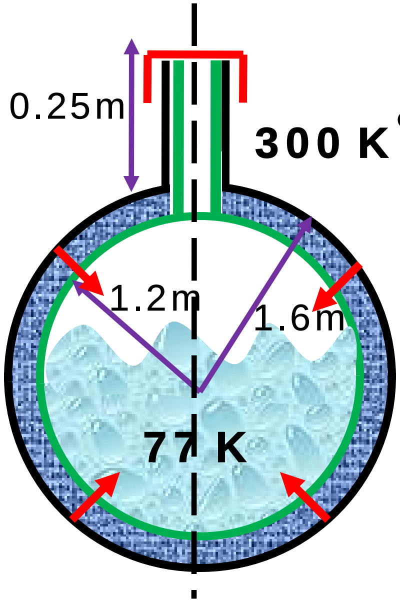
$$Q = F_e F_{1\rightarrow 2}\sigma A_1\left(T_2^4 - T_1^4\right)$$

$$F_e = 0.72$$

$$Q = (0.72)(1)(5.67)\left(10^{-8}\right)\pi\left(1.6^2\right)\left(300^4 - 77^4\right)$$

$$Q_r = 2648W$$

# Tutorial




Less Vacuum (1.5mPa)

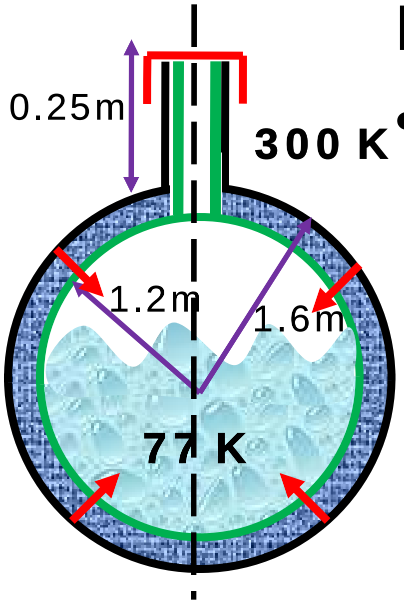
- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $T_1$=77 K, $T_2$=300 K, p=1.5 mPa.

$$\lambda = \frac{\mu}{p}\left(\frac{\pi RT}{2}\right)^{0.5}$$

$$\lambda = \frac{(18.47)(10^{-6})}{(1.5)(10^{-3})}\left(\frac{\pi(287.6)(300)}{2}\right)^{0.5} = 4.53$$

- It is clear that the mean free path (**λ**) is greater than distance between the surfaces (0.4m).

# Tutorial




Less Vacuum (1.5mPa)

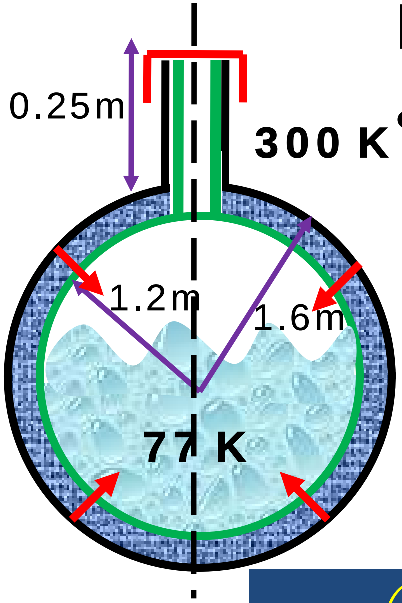
- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $T_1$=77 K, $T_2$=300 K, p=1.5 mPa.

$$F_a = \left( \frac{1}{\alpha_1} + \left( \frac{A_1}{A_2} \right) \left( \frac{1}{\alpha_2} - 1 \right) \right)^{-1}$$

$$F_e = \left( \frac{1}{1} + \left( \frac{1.2}{1.6} \right)^2 \left( \frac{1}{0.85} - 1 \right) \right)^{-1} = 0.91$$

| T (K) | Air |
|---|---|
| 300 | 0.8-0.9 |
| 78 | 1.0 |
| 20 | 1.0 |

# Tutorial




Less Vacuum (1.5mPa)

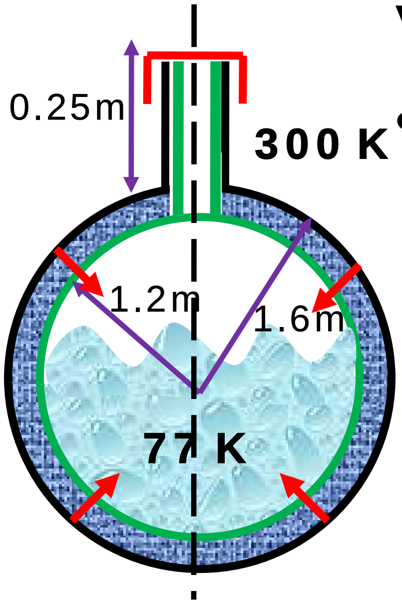
- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $T_1$=77 K, $T_2$=300 K, p=1.5 mPa.

$$\dot{Q} = \left( \left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{R}{8\pi T}\right)^{0.5} F_a \right) pA(T_2 - T_1)$$

$$\dot{Q} = \left( \left(\frac{1.4+1}{1.4-1}\right)\left(\frac{287.6}{8\pi(300)}\right)^{0.5} (0.91) \right)(1.5)\left(10^{-3}\right)(300-77)$$

$$Q_{gc} = 0.356W$$

# Tutorial




Vacuum alone

- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $k_A$, $T_1$=77K, $T_2$=300K, $e_1$, $e_2$=0.8, $F_{1\rightarrow 2}$=1.

$$Q = F_e F_{1\rightarrow 2} \sigma A_1 \left(T_2^4 - T_1^4\right)$$

$$F_e = 0.72$$

$$Q = (0.667)(1)(5.67)\left(10^{-8}\right)\pi\left(1.6^2\right)\left(300^4 - 77^4\right)$$

$$Q = 2648W$$

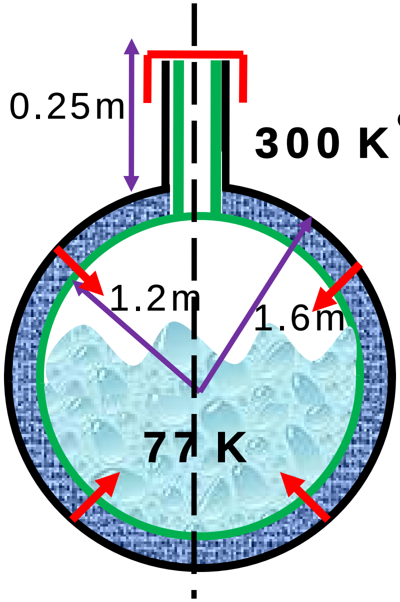
Vacuum + 10 shields

- $e_1$, $e_2$=0.8, $e_s$=0.05.

$$F_e = 0.003$$

$$Q = 11.02W$$

# Tutorial




Evacuated Fine Perlite ($k_A$ = 0.95mW/mK)

- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $k_A$, ΔT=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = 12.7W$$

50/50 Cu – Santocel ($k_A$ = 0.33mW/m-K)

- **Sphere** - $R_1$=1.6m, $R_2$=1.2m, $k_A$, ΔT=(300-77)=223.

$$Q = \frac{4\pi k_A R_1 R_2 \Delta T}{\left(R_2 - R_1\right)}$$

$$Q = 4.41W$$

# Tutorial

| Heat in leak (Q) | |
|---|---|
| Perlite | 349.7 W |
| Less Vacuum (1.5mPa) | $Q_r$=2648 W<br>$Q_{gc}$=0.356 W |
| Vacuum alone | 2648 W |
| Vacuum + 10 shields | 11.02 W |
| Evacuated Fine Perlite | 12.7 W |
| 50/50 Cu – Santocel | 4.41 W |

# Summary

- In vacuum, the radiation is the dominant mode of heat transfer.
- Evacuated powders are superior in performance than vacuum alone in **300-77 K**, as the radiation heat transfer is comparatively less.
- At low pressures and temperatures, the solid conduction in evacuated powder dominates the radiant heat transfer.
- In an opacified powder, the radiation heat transfer is minimized by addition of reflective flakes.

# Thank You!