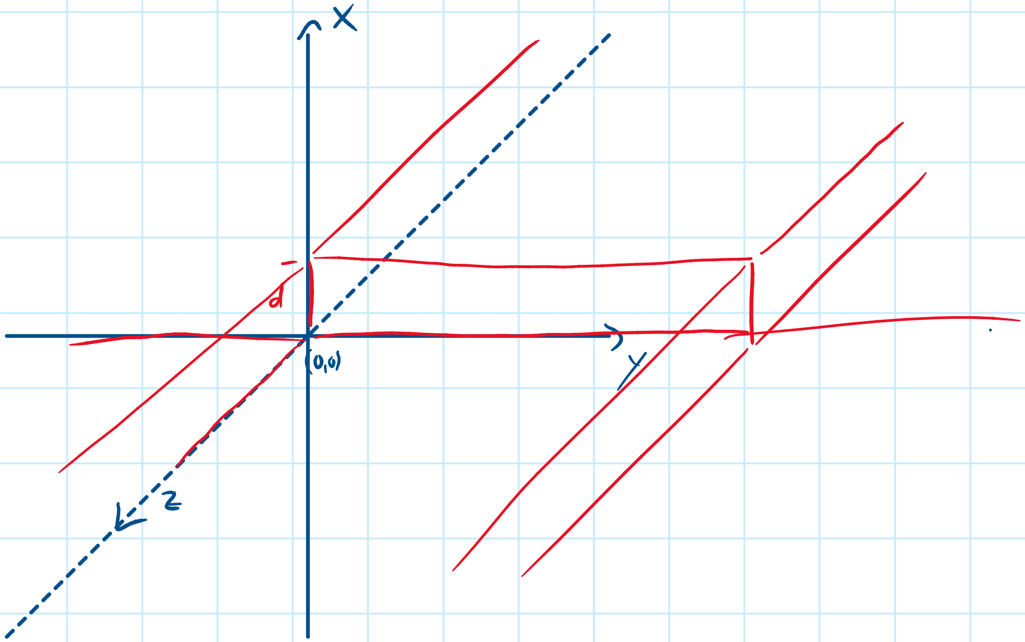
Donnerstag, 19. Juli 2018 01:12

Potential einer Platte
mit Ladungsdichte ϱ_0



$$\phi(\underline{r}) = \frac{\varrho_0}{4\pi\varepsilon_0} \int \frac{1}{|\underline{r} - \underline{r}'|} \, d^3 r'$$

Da nur x-Position entscheidend
ist $\quad r := \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \quad r := \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

Bsp

$r := \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} \qquad r := \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$

$$\phi(x) = \frac{S}{4\pi\varepsilon_0} \int_0^\infty \int_0^\infty \int_0^d \frac{1}{\sqrt{(x-x')^2 + y'^2 + z'^2}} \, dx' dy' dz'$$

$$= \underbrace{\frac{S}{4\pi\varepsilon_0}}_{k} \int_0^\infty \int_0^\infty \left[\ln\left(\sqrt{(x-x')^2 + y'^2 + z'^2} + x' - x \right) \right]_0^d dy' dz'$$

$$= k \int_0^\infty \int_0^\infty \left(\ln \sqrt{(x-d)^2 + y'^2 + z'^2} + d - x - \ln\left(\sqrt{x^2 + y^2 + z^2} - x \right) \right) d_y' d_z'$$

$$= k \int_0^\infty \int_0^\infty \ln\left(\frac{\sqrt{(x-d)^2 + y'^2 + z'^2} + d - x}{\sqrt{x^2 + y^2 + z^2} - x} \right) dy' dz'$$