Theoretical and practical aspects for balancing crankshafts of six-cylinder V60 degree engines

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Abstract: Owing to manufacturing variations, a counterweighted crankshaft still needs to be dynamically balanced on a balance machine before being put into service. To balance a non-symmetric counterweighted crankshaft, disconnected from the piston and connecting rod assemblies, on a balance machine, it is necessary to attach correct ring weights on crankpins to represent the dynamic effect of the piston and connecting rod assemblies. The central issues are: What should be the sizes and the locations of the ring weights? This paper presents a set of formulae for determining the ring weights needed for balancing crankshafts of six-cylinder V60 degree engines. The paper also discusses practical considerations, such as the design, manufacturing and installation of the ring weights, the method of testing, and subsequent adjustment if needed.

Keywords: engine; crankshaft; balance; ring weights; vibration; V60 degree engine; rotating mass; reciprocating mass.

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1 Introduction

Engine balancing to reduce, or preferably to eliminate, unwanted vibrations has been a long-time and ongoing effort of engine designers and vehicle manufacturers. References (Kuns and Plumridge, 1946; Timoshenko, 1948; Judge, 1936; Angle, 1925; Holowenko, 1955; Seaholm, 1924; Anderson, 1924; Robert, 1924; Platt, 1924; Smith, 1946; Wilson, 1929; Thomson, 1978; Heisler, 1995; Liu and Orzechowski, 2005; Buchholz, 2003; Burrell and Butler, 1954; Swan, 1922; Weertman and Lecher, 1970) summarise some of these works. The focus of these efforts is the crankshaft – often called the 'backbone of the engine' (Kuns and Plumridge, 1946). Vibration problems of engines generally include crankshaft vibrations. Indeed forces and moments generated by crankshaft imbalance are the major source of engine vibration.

To eliminate imbalance, a counter-weighted crankshaft must be balanced both statically and dynamically. A non-symmetrical counterweighted crankshaft, disconnected from the piston and connecting rod assemblies, however, cannot be directly balanced on a balance machine. That is, we cannot spin the crankshaft on a balance machine and balance it, since this condition is totally different from the operating condition in the engine. Therefore, in practice to balance the crankshaft on a balancing machine, it is necessary to add weights ('ring weights') to the crankpins to represent the dynamic effects of the pistons and connecting rods. Once the balancing process is completed, the ring-weights are removed and the crankshafts are ready to be put into service.

In this paper we consider the problem of crankshaft balancing of six-cylinder V-60 type internal combustion engines.

Although there is a wealth of information available on crankshaft balancing of various types of engines, very little appears to be known about balancing crankshafts of six-cylinder V-60 engines. In our analysis, therefore, we initially develop a theoretical basis for the use of ring weights. We then consider the design and manufacture of the weights. Finally we discuss testing, evaluation, and adjustments needed to obtain smooth-running engines.

The organisation of the paper is divided into four parts with the first of these determining the inertia forces in single-cylinder engines. The next two parts then extend these results to six-cylinder V-type engines. We then present formulas for finding the sizes and locations of the ring weights. The final part discusses testing and evaluation and then provides concluding remarks.

2 Inertia forces in a single-cylinder engine

In preparation for our analysis of six-cylinder V-type engines it is helpful to initially consider a single cylinder engine.

The principal parts of a single cylinder engine are:

1 the cylinder itself

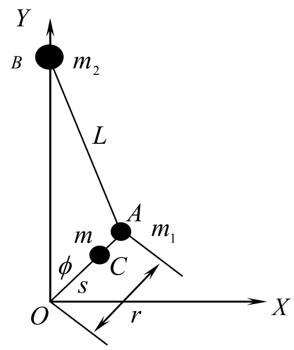
2 the piston

3 the crankshaft

4 the connecting rod.

The cylinder is usually stationary while the other three parts are moving. The piston and the connecting rod periodically accelerate and decelerate while the crank is normally rotating at a nearly uniform rate, but the mass centre of the crank is usually somewhat off the axis of rotation. The movements of the crank, the connecting rod, and the piston, thus produce inertia forces proportional to their masses and accelerations/decelerations.

Figure 1 A single cylinder engine system represented by three mass particles



We can reasonably model the moving parts by point masses as in Figure 1. In the figure and in the sequel we use the notation:

r	crank radius (1/2 stroke)
L	length of connecting rod
s	OC = distance to centre of gravity of crank
e	piston pin offset
m	mass of crank and crankpin
m_1	rotating mass of connecting rod which is equal to its big-end mass
m_2^*	small end mass of connecting rod
m_2^{**}	total mass of piston, piston pin, piston pin locking device, piston rings and oil rings
ϕ	angle of crank to cylinder axis, positive clockwise

$m^* = m_1 + m_2^*$ total mass of connecting rod

$m_2 = m_2^* + m_2^{**}$ reciprocating mass.

We will also introduce other specific notation as needed.

Following Timoshenko's procedure (Timoshenko, 1948) we model the connecting rod by two equivalent masses m_1 and m_2^* at the crank pin and at the gudgeon pin, respectively. These two masses are called the 'large-end mass' and the 'small-end mass'. Their sum must be equal to the mass of the connecting rod itself, and their relative magnitudes must be such that their mass centre coincides with that of the rod.

At the small end mass (point *B*), we add the masses of the piston, the piston pin, the piston pin locking devices, the piston rings, the spacer and the oil rings. We then denote the total mass at *B* as: m_2, and designate it as the 'reciprocating mass'. The mass of the crank and crank pin is *m* and it is assumed to be concentrated at *C*. With this modelling, the entire moving system is represented by three particles *A*, *B*, and *C* with masses m_1, m_2, and *m*, respectively.

Modelling the connecting rod by two concentrated end masses is not precise since the central moment of inertia of the rod in the axial direction is different than that of the two-mass system (Timoshenko, 1948; Holowenko, 1955; Platt, 1924; Wilson, 1929). Although there are more accurate models (Timoshenko, 1948; Platt, 1924) the difference between the resulting inertia torques of the rod and the two point masses is small. For heavy-oil engines the two-mass model is reasonably accurate and it is generally adopted by most analysts.

Since the system is symmetrical in the central plane normal to the axis, it is convenient to introduce coordinate axes: *X* and *Y*, with origin *O* in the central plane, as shown in Figure 1. Due to the symmetry the system may be represented by a single force passing through *O*, having *X* and *Y* components: $\sum F_X^e$ and $\sum F_Y^e$ together with a couple with torque: $\sum\left(F_Y^e x - F_X^e y\right)$.

We are primarily interested in the inertia forces generated by the moving system. Since the combination of the inertia forces and the externally applied forces form a zero force system, we can express the components of the equivalent inertia force through *O* as:

$$F_X = -\sum F_X^e \quad \text{and} \quad F_Y = -\sum F_Y^e \tag{1}$$

When the motion of the system is known, we can readily obtain the inertia force components as time rates of change of the linear momentum of the system. Specifically, by observing that the speeds of *C*, *A*, and *B* are: $s\dot{\phi}$, $r\dot{\phi}$ and $\dot{y}$, respectively (see Figure 1), and by projecting these speeds in the *X* and *Y* directions, the momentum principle provides the expressions:

$$\frac{d}{dt}\left(ms\dot{\phi}\cos\phi + m_1 r\dot{\phi}\cos\phi\right) = \sum F_X^e = -F_X \tag{2}$$

and

$$\frac{d}{dt}\left(-ms\dot{\phi}\sin\phi - m_1 r\dot{\phi}\sin\phi + m_2\dot{y}\right) = \sum F_Y^e = -F_Y \tag{3}$$

Then by carrying out the indicated differentiations we have:

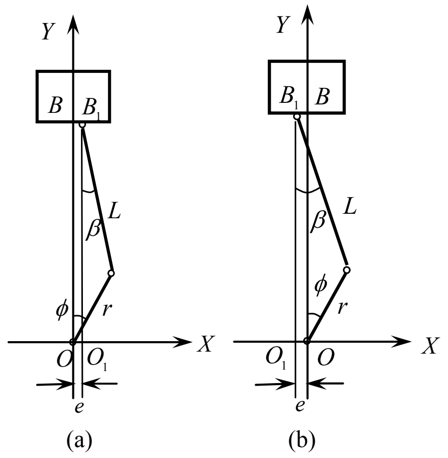
$$F_X = \left(ms + m_1 r\right)\left(\dot{\phi}^2 \sin\phi - \ddot{\phi}\cos\phi\right) \tag{4}$$

and

$$F_Y = \left(ms + m_1 r\right)\left(\dot{\phi}^2 \cos\phi + \ddot{\phi}\sin\phi\right) - m_2 \ddot{y} \tag{5}$$

Observe in the right sides of equations (4) and (5) all of the terms, except for: $-m_2\ddot{y}$ are expressions involving the crank angle ϕ. It happens that $\ddot{y}$ may also be expressed in terms of ϕ. To see this, consider a general case where there is a piston pin offset, denoted by *e*, as shown in Figure 2. The piston pin offset is classified into two categories (Angle, 1925; Holowenko, 1955; Kuns and Plumridge, 1946; Seaholm, 1924; Anderson, 1924; Robert, 1924; Platt, 1924; Smith, 1946; Wilson, 1929; Thomson, 1978; Heisler, 1995; Liu and Orzechowski, 2005; Buchholz, 2003; Burrell and Butler, 1954; Swan, 1922): One is the piston pin offset from the piston centre line towards the minor thrust side [see Figure 2(a)], where the crank rotates clockwise while the piston pin is located on the right side of the cylinder's centre line. The other is the piston pin offset from the piston centre line towards the major thrust side, that is, the crank rotates clockwise while the piston pin is located on the left side of the cylinder's centre line [see Figure 2(b)].

Figure 2 A single-cylinder engine with piston pin offset, (a) the pin offset towards minor thrust side (b) the pin offset towards major thrust side



A principal step in our analysis is the development of an expression for the piston position in terms of the crank angle ϕ. To simplify the analysis, the piston position is measured from the crank centre instead of from the topmost position.

Referring to Figure 2, we see that the distance O_1B_1 is:

$$O_1B_1 = r\cos\phi + L\cos\beta \tag{6}$$

Also, from Figure 2 we have:

$$L\sin\beta + e = r\sin\phi \tag{7}$$

If the piston pin offset e is away from the piston centre line toward the minor thrust side, as in Figure 2(a), then e is positive. Conversely, if the piston pin offset is away from the piston centre line toward the major thrust side, as in Figure 2(b), then e is negative. With no offset, e is zero.

From equation (7), $\sin\beta$ and $\cos\beta$ are immediately seen to be:

$$\sin\beta = \frac{r\sin\phi - e}{L} \tag{8}$$

and

$$\cos\beta = \sqrt{1-\left(\frac{r\sin\phi - e}{L}\right)^2} \tag{9}$$

Finally, by substituting from (9) into equation (6) we obtain:

$$O_1B_1 = r\cos\phi + L\sqrt{1-\left(\frac{r\sin\phi - e}{L}\right)^2} \tag{10}$$

In practical applications, r and e are usually small compared with L. Thus, by using the binomial theorem we can expand the radical of equation (10) into a rapidly converging power series. That is:

$$y = O_1B_1 = r\cos\phi + r\frac{e}{L}\sin\phi - \frac{r^2}{2L}\sin^2\phi + L\left(1-\frac{e^2}{2L^2}\right)+\ldots \tag{11}$$

where the last term being a constant, will not contribute to the inertia force components.

Finally, by substituting from equation (11) into equation (5) and differentiating, the force component F_Y may be expressed to the first order in e/L as:

$$\begin{aligned} F_Y &= \left(ms + m_1 r\right)\left(\dot{\phi}^2\cos\phi + \ddot{\phi}\sin\phi\right) + \\ &\quad + m_2 r\left[\dot{\phi}^2\left(\cos\phi + \frac{e}{L}\sin\phi\right) + \ddot{\phi}\left(\sin\phi - \frac{e}{L}\cos\phi\right)\right] \end{aligned} \tag{12}$$

The horizontal force component F_X given by equation (4), and repeated here is:

$$F_x = \left(ms + m_1 r\right)\left(\dot{\phi}^2\sin\phi - \ddot{\phi}\cos\phi\right) \tag{13}$$

In studying engine balance, we are most interested in the case where the engine runs with uniform angular speed ω. For this condition, in equations (12) and (13) the terms containing $\ddot{\phi}$ vanish. Then, by letting $\dot{\phi} = \omega$, we have:

$$F_X = \left(ms + m_1 r\right)\omega^2\sin\phi \tag{14}$$

and

$$F_Y = \left(ms + m_1 r\right)\omega^2\cos\phi + m_2\omega^2 r\left[\cos\phi + \frac{e}{L}\sin\phi\right] \tag{15}$$

3 Inertia forces and couples of a six-cylinder V-60 degree engine

We are now ready to consider the six-cylinder V-60 engines. These engines have two banks, each with three cylinders, and oriented at 60 degrees relative to each other. The crankshaft has six offsets (or 'throws') attached to the six connecting rods (three on each bank).

Unfortunately there is no commonly accepted procedure for numbering and labelling the cylinders. Therefore, to proceed, we arbitrarily adopt the nomenclature of the Bosch Handbook (Bosch, 1986). Specifically, the cylinders on the right bank (looking from behind the engine) are numbered: $C1$, $C2$, and $C3$ and then those on the left as: $C4$, $C5$, and $C6$ where $C1$ and $C4$ are in the front, farthest away from the power output side, as represented in Figure 3 (observe that the numbering scheme does not affect the results of the analysis).

Figure 3 Numbering of cylinders of a V6 engine

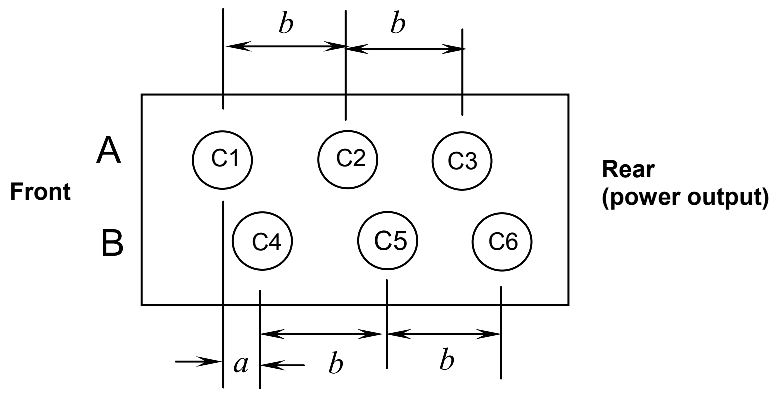
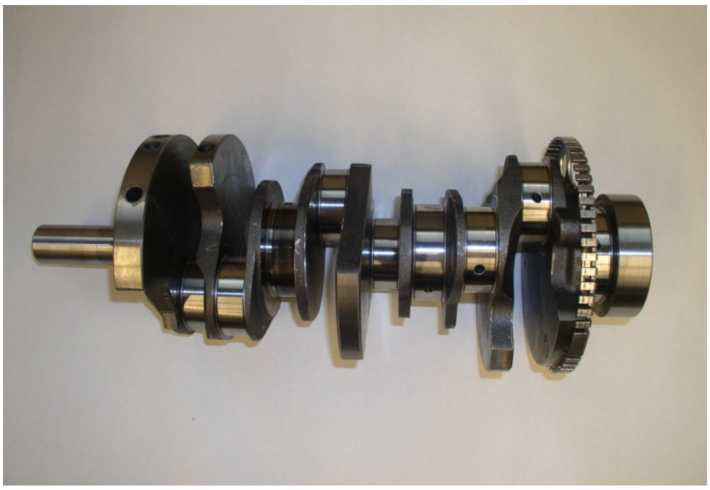


Figure 4 A counterweighted crankshaft of a six-cylinder V60 degree engine (see online version for colours)



Source: Photo Courtesy of Raven Engineering, Inc.

Figure 4 provides a photographic representation of a V-6 crankshaft and Figure 5 shows the crank arrangement where the cranks of *C*1, *C*2, and *C*3 are set at 180 degrees relative to the cranks of *C*6, *C*4, and *C*5, respectively.

Figure 5 Crank arrangement of the six-cylinder V-60 engine

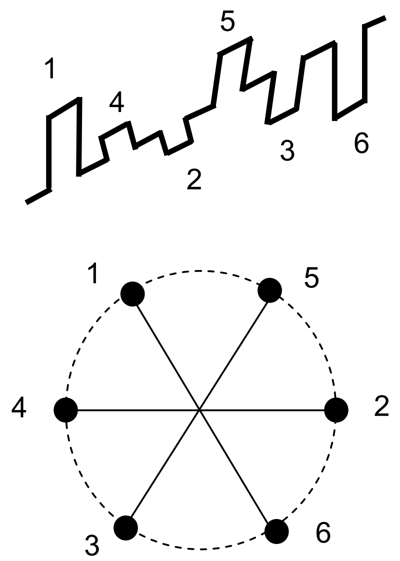
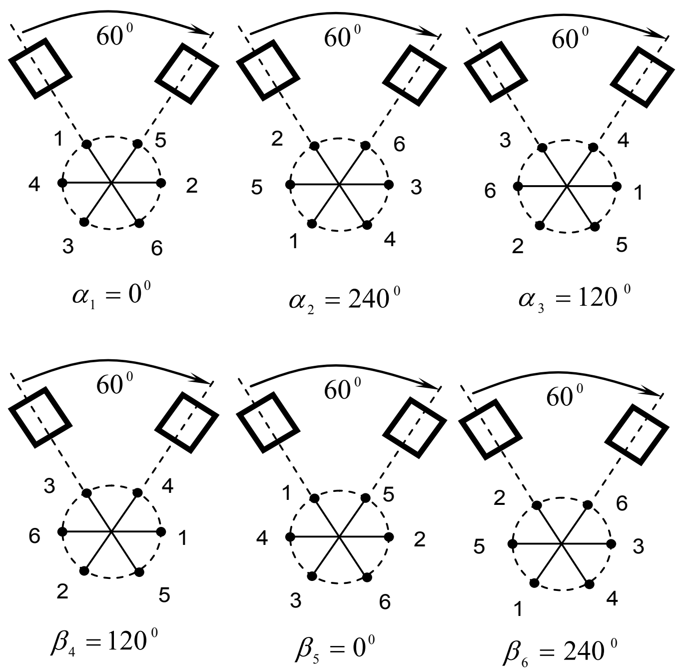


Figure 6 The phase angles between the six cylinders



We can use our single cylinder analysis to determine the inertia forces of the V-6 engines: To this end, it is useful to identify the phase angles of the six cylinders. Using *C*1 as a reference, and letting the crank angle of *C*1 be zero, Figure 6 shows the crank angles of the other five cylinders. Table 1 provides a listing of these angles.

Table 1 A listing of the phase angles between the six cylinders

Cylinder no. in Bank A	1	2	3
Phase angle symbol	α_1	α_2	α_3
Phase angle value	0°	240°	120°
Cylinder no. in Bank B	4	5	6
Phase angle symbol	β_4	β_5	β_6
Phase angle value	120°	0°	240°

By using equations (14) and (15) as a guide, we can develop the inertia force components for the cylinders as follows:

- in Bank A:

$$F_i^x = m_1\omega^2 r\sin(\phi-\alpha_i), (i=1,2,3) \tag{16}$$

$$F_i^y = m_1\omega^2 r\cos(\phi-\alpha_i) + m_2\omega^2 r\left[\cos(\phi-\alpha_i)+\frac{e}{L}\sin(\phi-\alpha_i)\right], (i=1,2,3) \tag{17}$$

- in Bank B:

$$F_j^x = m_1\omega^2 r\sin(\phi-\beta_j), (j=4,5,6) \tag{18}$$

$$F_j^y = m_1\omega^2 r\cos(\phi-\beta_j) + m_2\omega^2 r\left[\cos(\phi-\beta_j)+\frac{e}{L}\sin(\phi-\beta_j)\right], (j=4,5,6) \tag{19}$$

From these expressions and the phase angles listed in Table 1, we obtain the relations:

$$F_1^x = F_5^x,\ F_1^y = F_5^y \tag{20}$$

$$F_2^x = F_6^x,\ F_2^y = F_6^y \tag{21}$$

$$F_3^x = F_4^x,\ F_3^y = F_4^y \tag{22}$$

Substituting for the phase angles from Table 1 into equations (16) to (19), we obtain the expressions for the inertia forces as:

$$F_1^x = F_5^x = m_1\omega^2 r\sin\phi \tag{23}$$

$$F_1^y = F_5^y = (m_1+m_2)\omega^2 r\cos\phi + m_2\omega^2 r(e/L)\sin\phi \tag{24}$$

$$F_2^x = F_6^x = -m_1\omega^2 r\left[\frac{1}{2}\sin\phi - \frac{\sqrt{3}}{2}\cos\phi\right] \tag{25}$$

$$F_2^y = F_6^y = -\left(m_1 + m_2\right)\omega^2 r\left[\frac{1}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right] - m_2\omega^2 r(e/L)\left[\frac{1}{2}\sin\phi - \frac{\sqrt{3}}{2}\cos\phi\right] \tag{26}$$

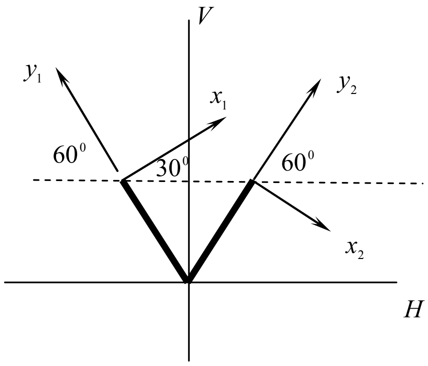
$$F_3^x = F_4^x = -m_1\omega^2 r\left[\frac{1}{2}\sin\phi + \frac{\sqrt{3}}{2}\cos\phi\right] \tag{27}$$

$$F_3^y = F_4^y = -(m_1 + m_2)\omega^2 r\left[\frac{1}{2}\cos\phi - \frac{\sqrt{3}}{2}\sin\phi\right] - m_2\omega^2 r(e/L)\left[\frac{1}{2}\sin\phi + \frac{\sqrt{3}}{2}\cos\phi\right] \tag{28}$$

From these expressions, we see that:

$$\left(F_1^x + F_2^x + F_3^x\right) = 0 \text{ and } \left(F_1^y + F_2^y + F_3^y\right) = 0 \tag{29}$$

Figure 7 Coordinate systems



Observe in Figure 7 that for the six-cylinder V-60 engine, the angle between the banks is 60 degrees. Consequently the banks are inclined at 60 degrees relative to the horizontal, and at 30 degrees relative to the vertical. The horizontal components of the inertia forces on the system are then:

- for cylinders in Bank A:

$$F_j^H = F_j^x \cos 30° - F_j^y \cos 60°, (j = 1,2,3) \tag{30}$$

- for cylinders in Bank B:

$$F_j^H = F_j^x \cos 30° + F_j^y \cos 60°, (j = 4,5,6) \tag{31}$$

Similarly, the vertical components of the inertia forces are:

- for cylinders in Bank A:

$$F_j^V = F_j^x \sin 30° + F_j^y \cos 60°, (j = 1,2,3) \tag{32}$$

- for cylinders in Bank B:

$$F_j^V = -F_j^x \sin 30° + F_j^y \sin 60°, (j = 4,5,6) \tag{33}$$

By inspection of the various terms of equations (30) to (33) we see that:

$$\sum_{j=1}^{6} F_j^H = 2\left(F_1^x + F_2^x + F_3^x\right)\cos 30° = 0 \tag{34}$$

$$\sum_{j=1}^{6} F_j^V = 2\left(F_1^y + F_2^y + F_2^y\right)\sin 60° = 0 \tag{35}$$

Equations (34) and (35) show that with the crank arrangement of Figure 5, the horizontal and vertical components of the inertia forces are self-balanced. Observe, however, in Figures 3, 4 and 5, the crankshaft is not symmetric when viewed along axes perpendicular to the rotation axis. Consequently, the inertia forces create rotating moments about these axes.

To see this in greater detail, consider the moment of the inertia forces about the axis of cylinder C1: from Figure 3, we see after routine analysis, that the horizontal and vertical components of the inertia moments are:

$$M_H = F_2^V b + F_3^V (2b) + F_4^V a + F_5^V (a+b) + F_6^V (a+2b) \tag{36}$$

$$M_V = -\left[F_2^H b + F_3^H (2b) + F_4^H a + F_5^H (a+b) + F_6^H (a+2b)\right] \tag{37}$$

Substituting the expressions of the forces from equations (31) to (34) into equations (36) and (37), we obtain:

$$\begin{aligned} M_H = & -\sqrt{3} b m_1 \omega^2 r \left[\frac{\sqrt{3}}{2}\sin\phi + \frac{3}{2}\cos\phi\right] - \frac{\sqrt{3}}{2} b m_2 \omega^2 r \left[\frac{3}{2}\cos\phi + \frac{\sqrt{3}}{2}\sin\phi\right] \\ & - \frac{\sqrt{3}}{2} b m_2 \omega^2 r \frac{e}{L}\left[\frac{3}{2}\sin\phi - \frac{\sqrt{3}}{2}\cos\phi\right] \end{aligned} \tag{38}$$

$$\begin{aligned} M_V = & \sqrt{3} b m_1 \omega^2 r \left[\frac{3}{2}\sin\phi - \frac{\sqrt{3}}{2}\cos\phi\right] - \frac{3}{2} b m_2 \omega^2 r \left[\frac{1}{2}\cos\phi - \frac{\sqrt{3}}{2}\sin\phi\right] \\ & - \frac{3}{2} b m_2 \omega^2 r \frac{e}{L}\left[\frac{1}{2}\sin\phi + \frac{\sqrt{3}}{2}\cos\phi\right] \end{aligned} \tag{39}$$

Observe that the inertia torque components M_H and M_V of equations (38) and (39) are independent of the length parameter a (see Figure 3). This means that although the configuration of a V-60 six–cylinder engine may be different, the inertia moments are the same. For example, if cylinder C1 is in Bank B instead of Bank A, the expressions for inertia moments are unchanged.

Next, observe that, taken together, equations (38) and (39) describe a rotating unbalance inertia torque with constant magnitude given by:

$$M = \sqrt{(M_H)^2 + (M_V)^2} = \frac{3}{2}\sqrt{\left[(2m_1 + m_2)^2 + (m_2 e / L)^2\right]}\,\omega^2 rb \tag{40}$$

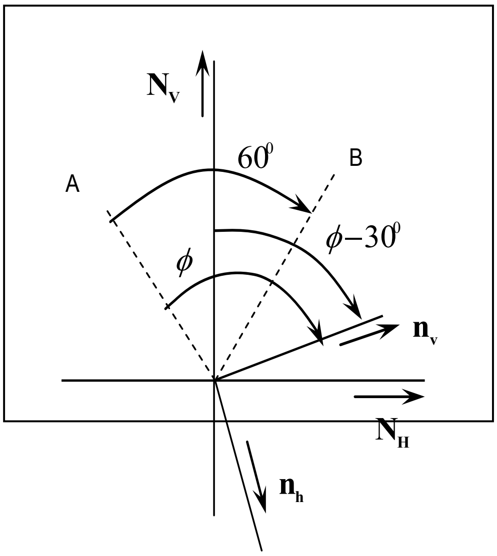
where the torque rotation is in the direction of the crank rotation and at the same rate.

To describe the inertia torque components in greater detail, it is helpful to introduce a pair of perpendicular unit vector sets N_H, N_V, n_h and n_v as in Figure 8. Let N_H and N_V be fixed relative to the engine block and parallel to horizontal and vertical directions respectively. Let n_h and n_v be fixed relative to the crankshaft, with n_v parallel to crank 1, directed radially outward. Let n_h and n_v be respectively parallel to N_H and N_V when the crank angle ϕ is 30°. Table 2 lists the corresponding direction cosines (Huston and Liu, 2001) for the two unit vector sets..

Table 2 The direct cosines for the two unit vector sets of Figure 8

	N_V	N_H
n_v	$\cos(\phi - 30°)$	$\sin(\phi - 30°)$
n_h	$-\sin(\phi - 30°)$	$\cos(\phi - 30°)$

Figure 8 Two unit vector sets



By referring to Table 2, the components m_h^c and m_v^c of the inertia moments on the crankshaft are seen to be:

$$m_h^c = -M_V \sin(\phi - 30°) + M_H \cos(\phi - 30°) \tag{41}$$

$$m_v^c = M_V \cos(\phi - 30°) + M_H \sin(\phi - 30°) \tag{42}$$

By substituting from equations (38) and (39) for M_H and M_V into equations (41) and (42), we obtain (see Figure 9):

$$m_h^c = -3\left(m_1 + \frac{m_2}{2}\right)\omega^2 rb \tag{43}$$

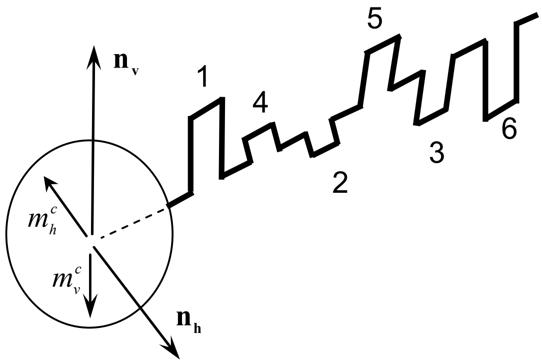
$$m_v^c = -\frac{3}{2}\frac{e}{L} m_2 \omega^2 rb \tag{44}$$

Also, the magnitude of the moment is:

$$M = \sqrt{\left(m_v^c\right)^2 + \left(m_h^c\right)^2} = \frac{3}{2}\sqrt{\left[\left(2m_1 + m_2\right)^2 + \left(m_2 e / L\right)^2\right]}\,\omega^2 rb \tag{45}$$

which as expected, is the same as in equation (40).

Figure 9 Couples of inertia forces expressed in the crankshaft coordinate system



It should be noted that equations (43) and (44) are valid when the pin offset is towards the minor thrust side. When the pin offset is towards the major thrust side, it is necessary to replace e with $-e$ in equations (43) and (44).

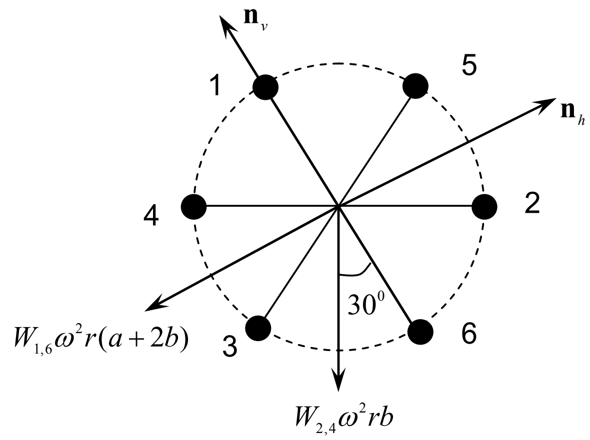
4 Determination of ring weights

From equations (43) and (44) we see that when the engine runs with uniform angular speed ω, the rotating mass m_1, the reciprocating mass m_2 and the pin offset e will produce two inertia couples. Since the inertia couples are independent of the position of the crank, it is possible to replace these inertia couples by using suitable ring weights fastened to each of the crankpins. To this end, we have the following options:

4.1 Option 1: use of four ring weights

Let two identical ring weights, each having mass $W_{1,6}$, be attached on crank 1 and 6, respectively. Next, let another two identical ring weights, having mass $W_{2,4}$, be fastened on crankpins 2 and 4, respectively. Thus when the crankshaft is rotating with a uniform angular speed ω, these ring weights will produce the inertia torques: $W_{1,6}\omega^2 r(a+2b)$ and $W_{2,4}\omega^2 rb$, as represented in Figure 10.

Figure 10 Inertial moments produced by ring weights on cranks 1, 6, 2 and 4



The purpose of these ring weights is to produce an equal inertia moment to that produced by the rotating mass, the reciprocating mass, and the pin offset. When the crankshaft is rotating with a uniform angular speed ω, we obtain the expressions:

$$-W_{2,4}\omega^2 rb\cos 30° = m_v^c \tag{46}$$

and

$$-W_{1,6}\omega^2 r(a+2b) - W_{2,4}\omega^2 rb\sin 30° = m_h^c \tag{47}$$

By substituting for m_h^c and m_v^c from equations (43) and (44) we find the masses of the ring weights to be:

$$W_{2,4} = \sqrt{3}\frac{e}{L}m_2 \tag{48}$$

and

$$W_{1,6} = \frac{1}{(2+a/b)}\left[3\left(m_1 + \frac{m_2}{2}\right) - \frac{\sqrt{3}}{2}\frac{e}{L}m_2\right] \tag{49}$$

In the special case where there is no offset ($e = 0$), we have:

$$W_{2,4} = 0 \tag{50}$$

and

$$W_{1,6} = \frac{3}{2+a/b}\left(m_1 + \frac{m_2}{2}\right) \tag{51}$$

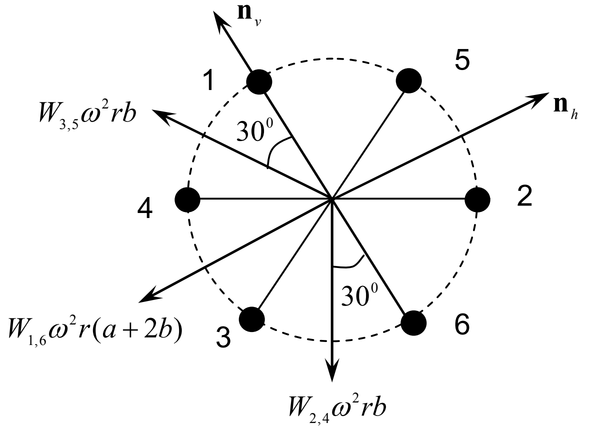
Equations (50) and (51) indicate that for the specific case of no offset ($e = 0$), only two identical ring weights, attached on crank 1 and 6, are needed to balance the crankshaft.

4.2 Option 2: use of six ring weights

It is the belief of the authors that some manufacturers use six ring weights attached to the crankpins during balancing the crankshafts of six-cylinder V60 degree engines on a balance machine. The mass of each ring weight used is simply equal to the sum of the rotating mass and half the reciprocating mass, that is $m_1 + m_2/2$. In the following paragraphs, we will show that this is not precise.

To this end, let two identical ring weights, having masses $W_{1,6}$, be attached to crankpins 1 and 6. Next, let another two identical weights, having masses $W_{2,4}$ be attached to crankpins 2 and 4. Finally let two identical weights, having masses $W_{3,5}$ be attached to crankpins 3 and 5. Then when the crankshaft is rotating with a uniform angular speed ω, the weight pairs will produce the inertia torques: $W_{1,6}\omega^2 r(a+2b)$, $W_{2,4}\omega^2 rb$ and $W_{3,5}\omega^2 rb$, as represented in Figure 11.

Figure 11 Inertia moments produced by six ring weights



As before, the requirement for the ring weights is that with the crankshaft rotating at a uniform rate, the inertia moments created by the ring weights will be the same as that crated by rotating mass, the reciprocating masses, and the pin offset. This leads to the expressions:

$$W_{3,5}\omega^2 rb\cos 30° - W_{2,4}\omega^2 rb\cos 30° = m_v^c \tag{52}$$

$$-W_{1,6}\omega^2 r(a+2b) - W_{2,4}\omega^2 rb\sin 30° - W_{3,5}\omega^2 rb\sin 30° = m_h^c \tag{53}$$

Observe in equations (52) and (53), there are three unknown ring weight masses: $W_{1,6}$, $W_{2,4}$ and $W_{3,5}$. But with only two equations we cannot determine unique values for these masses. We can, however, make some reasonable assignments for the values. For example, we can take $W_{1,6}$ and $W_{2,4}$ to be identical ($W_{1,6} \equiv W_{2,4}$). With this assumption and by substituting for m_h^c and m_v^c from equations (43) and (44) into equations (52) and (53), we then obtain:

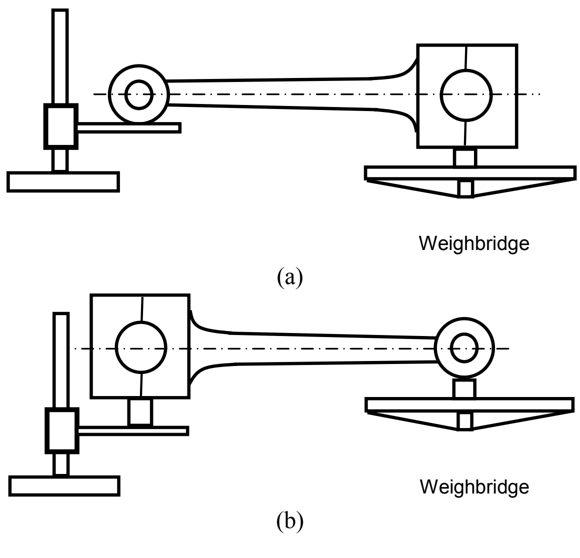
$$W_{1,6} = W_{2,4} = \frac{1}{3 + a/b}\left[3\left(m_1 + \frac{m_2}{2}\right) + \frac{\sqrt{3}}{2}\frac{e}{L}m_2\right] \tag{54}$$

$$W_{3,5} = W_{2,4} - \frac{\sqrt{3}e}{L}m_2 \tag{55}$$

If the terms containing (e/L) are ignored, the six ring weight masses become the same as having the value:

$$W_{1,6} = W_{2,4} = W_{3,5} = \frac{3}{3 + a/b}\left(m_1 + \frac{m_2}{2}\right) \tag{56}$$

Figure 12 Experimental setup for weighing connecting rod mass, (a) for big end mass (b) for small end mass



5 Design and manufacture of ring weights

In view of our findings and results we can make the following observations:

First, to accurately determine the ring weights, the rotating and reciprocating masses must be accurately determined. For the connecting rod, its rotating (large-end) and the reciprocating (small-end) masses can be determined experimentally using a scale and a special fixture as in Figure 12 (Wilson, 1929). That is, each end of the rod is weighted in turn, with the other end suspended horizontally. The result may be validated by adding together the rotating and reciprocating masses. The results should be equal to the total mass of the connecting rod. Since the weight of the components, as manufactured, varies, the rotating and reciprocating masses used in the formulae must be carefully determined by taking several complete sets of parts, and averaging their weights.

Next, the design and manufacture of the ring weights must satisfy the following requirements (Anderson, 1924; Liu and Orzechowski, 2005):

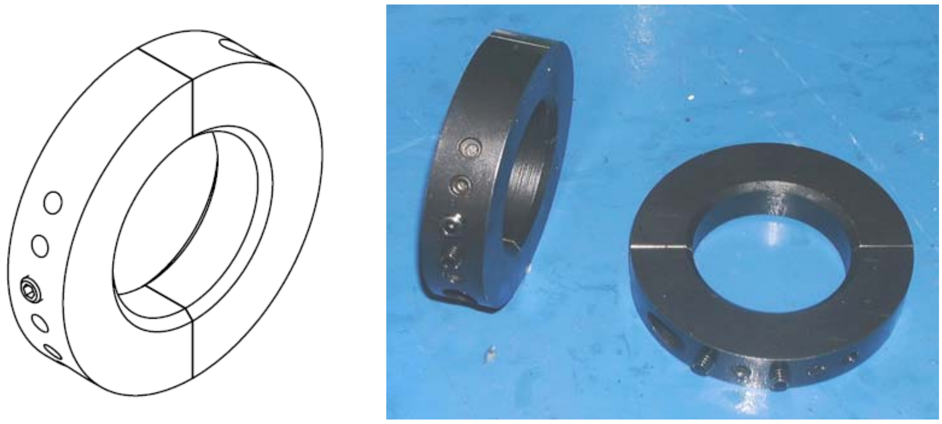
1 their weights must be precise. Otherwise, all advantage derived from balancing will be of no avail

2 the rings must be hardened to withstand wear and possible deformation

3 each ring weight itself must be balanced statically and dynamically.

Figure 13 shows a sketch and photo of a ring weight. Figure 14 shows a crankshaft balance machine. Figures 15 and 16 show installation procedure of the ring weights on the crankshaft which is to be balanced on a balance machine.

Figure 13 A ring weight, (a) a sketch (b) photo Courtesy of Raven Engineering, Inc. (see online version for colours)

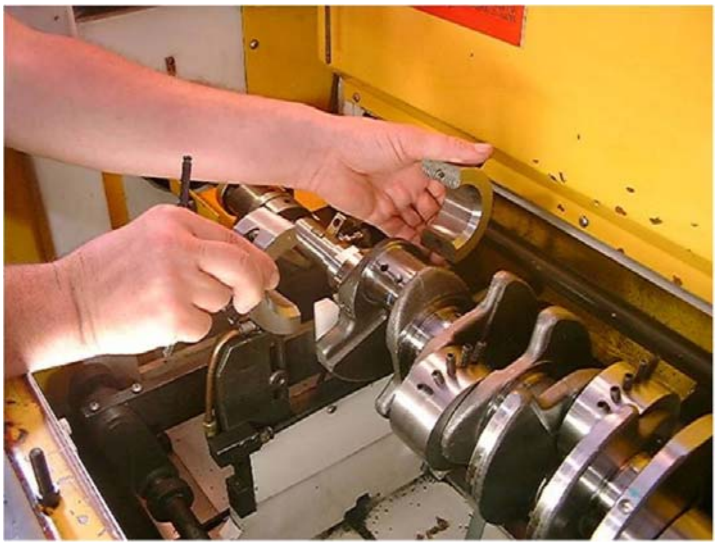


(a) (b)

Figure 14 A photo of crankshaft balance machine (see online version for colours)

Source: Photo Courtesy of Raven Engineering, Inc.

Figure 15 Ring weight installation on crankshaft – Step 1 (see online version for colours)



Source: Photo Courtesy of Raven Engineering, Inc.

Figure 16 Ring weight installation on crankshaft – Step 2 (see online version for colours)

Source: Photo Courtesy of Raven Engineering, Inc.

6 Experimental results/conclusions

The objective of our effort is to balance the crankshafts of six-cylinder V60 degree engines. Table 3 lists the relevant parameters.

Table 3 Parameters for calculation of ring weights

Parts		*Mass (gram)*
1	Piston	355.9
2	Snap ring	1.8
3	Pin	105.6
4	Ring 1	8.1
5	Ring 2	7.8
6	Oil ring 2pc assembly	6.1
7	Rod –small end	113.25
Total reciprocating mass		598.55
8	Rod – large end	434.5
9	Rod bearing	35.6
10	Oil in connecting rod bearing cap	1.78
11	Oil in main-pin oil hole	1.03
12	Oil wetting	7.92
Total rotating mass		480.83
Distance a (mm)		40
Distance b (mm)		106.5
Rod length L (mm)		156.5.
Pin offset e (mm)		0.8

First, we consider use of four ring weights. Since the piston pin offset e is 0.8 mm and the connecting rod length L is 156.5 mm. The weight of ring weights $W_{2,4}$ are given by [see equation (48)]:

$$W_{2,4} = \frac{\sqrt{3}}{2}\frac{e}{L}m_2 = \frac{\sqrt{3}}{2}\frac{0.8}{156.5}\left(598.55\right) \approx 2.65 \text{ (gram)}$$

The weight of ring weights $W_{1,6}$ are given by [see equation (49)]:

$$W_{1,6} = \frac{1}{2+a/b}\left[3\left(m_1 + \frac{m_2}{2}\right) - \frac{\sqrt{3}}{2}\frac{e}{L}m_2\right]$$

$$= \frac{1}{2.3756}\left[3\left(480.83 + \frac{598.55}{2}\right) - \frac{\sqrt{3}}{2}\frac{0.8}{156.5}(598.55)\right] \approx 984.0 \text{ (gram)}$$

Since the ring weight $W_{2,4}$ is very small, it is reasonable to ignore the effect of piston pin offset on the ring weights. Therefore, we can simply use two ring weights (that is $W_{1,6}$) during balancing the crankshafts on a balance machine. Note that, for the crankshafts of eight-cylinder V90 degree engines, the effect of piston offset on the ring weights cannot be ignored (Liu and Orzechowski, 2005).

We use the two-plane influence coefficient balancing method for balancing the crankshafts. A detailed description of the method can be found in reference (Gunter and Jackson, 1999). In this method (see the Appendix), a trial mass is placed in the plane at one end of the crankshaft, and vibration responses at both ends are then recorded. From these measurements, two influence coefficients may be computed. The trial mass is removed and placed at the other end. The procedure is then repeated generating two additional influence coefficients. This then forms a 2 × 2 matrix of complex influence coefficients which must be inverted for the imbalance. This method can determine the imbalance magnitudes and relative phase angles on the two planes. To correct for dynamic imbalance, it is necessary to introduce on each plane a radial inertia force that is equal in magnitude but opposite in direction to the one manifested by the balancing machine. This is done, in practice, by removing an equal amount of material from the two end planes.

Our experience shows that with two ring weights as indicated above, when the crankshaft is balanced to a level which is less than 120 gram.mm on each end on a balance machine at 850 rpm, when the ring weights are removed and the crankshaft is put back into service, the residual engine vibration level is acceptable. The method presented herein has recently been successfully tested and applied in automotive engine design and manufacture.

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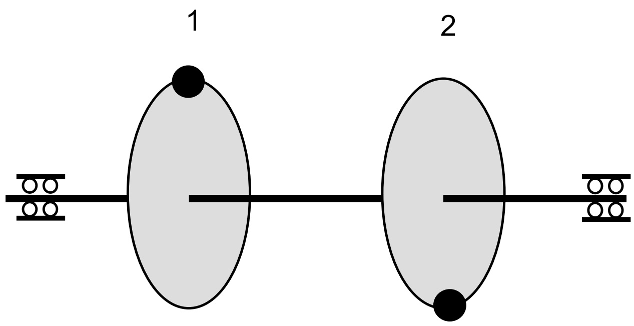
Appendix

The influence coefficient method – two plane balancing

A detailed description of the method can be found in reference (Gunter and Jackson, 1999). The basic assumption of this method is that vibration measured at a particular location at a fixed angular speed is the product of a linear combination of imbalance and sensitivity, or the 'influence coefficients'.

Figure A1 Two-plane unbalance system



Referring to Figure A1, the initial vibration readings $Z_1(\omega)$ and $Z_2(\omega)$ at the two planes for a fixed angular speed ω can be written as:

$$Z_1(\omega) = Z_1 e^{j\phi_1} \quad \text{and} \quad Z_2(\omega) = Z_2 e^{j\phi_2} \tag{A1}$$

where j is the imaginary unit and ϕ_1 and ϕ_2 are plane angles.

The vibrations are assumed to be linear combinations of the unknown unbalances U_1 and U_2 as follows:

$$\begin{bmatrix} Z_1(\omega) \\ Z_2(\omega) \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \tag{A2}$$

With the placement of a trial unbalance U_{t1} at plane 1, new vibrations may be recorded as:

$$\begin{bmatrix} Z_{11}(\omega) \\ Z_{21}(\omega) \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \begin{bmatrix} U_1 + U_{t1} \\ U_2 \end{bmatrix} \tag{A3}$$

where Z_{11} and Z_{21} are the vibrations recorded at planes 1 and 2 due to the trial unbalance placed in plane 1.

The influence coefficients S_{11} and S_{21} may then be computed as:

$$S_{11} = \frac{Z_{11} - Z_1}{U_{t1}},\ S_{21} = \frac{Z_{21} - Z_2}{U_{t1}} \tag{A4}$$

Next, the first trial unbalance weight is removed and a second trial unbalance weight U_{t2} is placed in the second plane. The resulting vibrations are:

$$\begin{bmatrix} Z_{12}(\omega) \\ Z_{22}(\omega) \end{bmatrix} = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 + U_{t2} \end{bmatrix} \tag{A5}$$

The influence coefficients S_{12} and S_{22} may then be computed as:

$$S_{12} = \frac{Z_{12} - Z_1}{U_{t2}},\ S_{22} = \frac{Z_{22} - Z_2}{U_{t2}} \tag{A6}$$

The unknown imbalance of the original system at planes 1 and 2, (U_1 and U_2), are given by:

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \tag{A7}$$

That is:

$$U_1 = \frac{Z_1 S_{22} - Z_2 S_{12}}{\Delta} \text{ and } U_2 = \frac{Z_2 S_{11} - Z_1 S_{21}}{\Delta} \tag{A8}$$

where

$$\Delta = S_{11} S_{22} - S_{12} S_{21} \tag{A9}$$