*Example 3.6*

The single index and the single argument express the first-order structure of these transition currents $J_\mu(x)$. According to Sect. 3.2, (3.51), each first-order transition current produces a vector potential

$$A^\mu(x) = \int \mathrm{d}^4y \; D_\mathrm{F}(x-y)\; J^\mu(y) \ .$$

Generalizing this relation we can conjecture that the following expression describes the electromagnetic fields produced by the proton:

$$\begin{aligned}
&A_\mu(x)A_\nu(y)\\
&\quad = \int \mathrm{d}^4X\,\mathrm{d}^4Y \; D_\mathrm{F}(x-X)D_\mathrm{F}(y-Y)J^{\mathrm{p}(2)}_{\mu\nu}(X,Y)\\
&\quad = \sum_{n;\,p_0>0} \int \mathrm{d}^4X\,\mathrm{d}^4Y \; D_\mathrm{F}(x-X)D_\mathrm{F}(y-Y)\big(J^\mathrm{p}_\mu(X)\big)_{fn}\big(J^\mathrm{p}_\nu(Y)\big)_{ni}\Theta(X_0-Y_0)\\
&\qquad - \sum_{n;\,p_0<0} \int \mathrm{d}^4X\,\mathrm{d}^4Y \; D_\mathrm{F}(x-X)D_\mathrm{F}(y-Y)\big(J^\mathrm{p}_\mu(X)\big)_{fn}\big(J^\mathrm{p}_\nu(Y)\big)_{ni}\Theta(Y_0-X_0)\\
&\quad = e_\mathrm{p}^2 \int \mathrm{d}^4X\,\mathrm{d}^4Y \; D_\mathrm{F}(x-X)D_\mathrm{F}(y-Y)\\
&\qquad \times \bar\psi^\mathrm{p}_f(X)\,\gamma_\mu \left( \sum_{n;\,p_0>0} \Theta(X_0-Y_0)\; \psi^\mathrm{p}_n(X)\; \bar\psi^\mathrm{p}_n(Y) \right.\\
&\qquad \left. - \sum_{n;\,p_0<0} \Theta(Y_0-X_0)\; \psi^\mathrm{p}_n(X)\; \bar\psi^\mathrm{p}_n(Y) \right) \gamma_\nu \psi^\mathrm{p}_i(Y)\\
&\quad = \mathrm{i}e_\mathrm{p}^2 \int \mathrm{d}^4X\,\mathrm{d}^4Y \; D_\mathrm{F}(x-X)D_\mathrm{F}(y-Y)\bar\psi^\mathrm{p}_f(X)\gamma_\mu S^\mathrm{p}_\mathrm{F}(X-Y)\gamma_\nu\psi^\mathrm{p}_i(Y) \ .
\end{aligned} \tag{6}$$

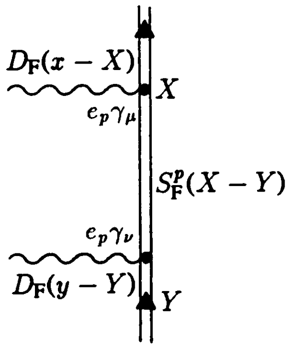


**Fig. 3.14.** The second-order proton transition current involves two photons. One propagates between the space–time points $x$ and $X$, the second one between $y$ and $Y$. Depending on the time ordering of the arguments of $D_\mathrm{F}(x-X)$ and $D_\mathrm{F}(y-Y)$ the photons are either absorbed or emitted. The propagation of the proton between the vertices at $X$ and $Y$ is described by the Feynman propagator $S^\mathrm{p}_\mathrm{F}(X-Y)$

Each single first-order transition current occurring in the products (5) creates a photon field like the one produced by the transition current in (3.51). Note the factor $+\mathrm{i}$ occurring in the last step that cancels the factor $-\mathrm{i}$ contained in the proton propagator $S^\mathrm{p}_\mathrm{F}(X-Y)$. Equation (6) can be substantiated by considering the differential equation for the two-photon field $A_{\mu\nu}(x,y)$ :

$$\Box_x \Box_y \, A_{\mu\nu}(x,y) = (4\pi)^2 \, J^{\mathrm{p}(2)}_{\mu\nu}(x,y) \ , \tag{7}$$

which is analogous to (3.42). The expression (6) obviously fulfils this differential equation. This result can be represented graphically as shown in Fig. 3.14.

The two photons are emitted at the space–time points $Y$, $X$. At each vertex a factor $e_\mathrm{p}\gamma_\mu$ or $e_\mathrm{p}\gamma_\nu$, respectively, enters into the calculation of the fields. The proton line between $Y$ and $X$ is called an *internal proton line*. It represents the propagation of the proton between the points of interaction with the photons according to the proton propagator $S^\mathrm{p}_\mathrm{F}(X-Y)$. The photons emitted by the proton at $Y$ and $X$ travel to the electron which absorbs them at the space–time points $y$