EXAMPLE

3.6 Higher-Order Electron–Proton Scattering

In Sect. 3.2 we calculated electron–proton scattering to lowest order in $\alpha = e^2$. Now we shall discuss corrections arising at the next higher order of the perturbation expansion. To that end we refer to the general expression of the n^{th} order contribution to the electron scattering matrix (2.44). The amplitude of second-order electron–proton interaction is given by

$$S_{fi}^{(2)} = -\mathrm{i}e^2 \int \mathrm{d}^4x\, \mathrm{d}^4y\, \bar{\psi}_f(x) \not{A}(x) S_\mathrm{F}(x-y) \not{A}(y) \psi_i(y) \ . \tag{1}$$

As in Sect. 3.2, the electromagnetic potential $A(x)$ is produced by the proton current. However, for us to be consistent the proton current has also to be treated in second order. To do this we again consider (1), describing the interaction of the electron current with the $A_\mu(x)\ A_\nu(y)$ fields in second order. Again, we require the total expression for $S_{fi}^{(2)}$ to be symmetric with respect to the electron and proton currents. From (1), it is obvious that the second-order electron current is given by

$$J_{\mu\nu}^{(2)}(x, y) = \ \mathrm{i}e^2\ \bar{\psi}_f(x)\ \gamma_\mu S_\mathrm{F}(x-y)\gamma_\nu\ \psi_i(y) \ . \tag{2}$$

The factor i here has been introduced for convenience. Namely, inserting the electron propagator $S_\mathrm{F}(x-y)$ explicitly this becomes

$$\begin{aligned} J_{\mu\nu}^{(2)}(x, y) = e^2 \bar{\psi}_f(x)\gamma_\mu \Bigg(& \sum_{n;\,p_0>0} \Theta(x_0-y_0)\psi_n(x)\bar{\psi}_n(y) \\ & - \sum_{n;\,p_0<0} \Theta(y_0-x_0)\psi_n(x)\bar{\psi}_n(y) \Bigg) \gamma_\nu \psi_i(y) \ . \end{aligned} \tag{3}$$

The two indices (and the two arguments x, y) indicate the second-order structure of the transition current. Here we used the Stückelberg–Feynman propagator (2.24). The factor i introduced in (2) cancels the factor minus i included in the propagator $S_\mathrm{F}(x-y)$, enabling us to represent the second-order current $J^{\mu\nu}(x, y)$ as a sum ($\sum_n$) of products consisting of first-order transition currents (cf. Sect. 3.2, (3.53)) of the form

$$\big(J_\mu(x)\big)_{fn} = e\ \bar{\psi}_f(x)\ \gamma_\mu\ \psi_n(x) \ , \tag{4}$$

which yields

$$\begin{aligned} J_{\mu\nu}^{(2)}(x, y) = & \sum_{n,\,p_0>0} \big(J_\mu(x)\big)_{fn} \big(J_\nu(y)\big)_{ni}\ \Theta(x_0-y_0) \\ & - \sum_{n,\,p_0<0} \big(J_\mu(x)\big)_{fn} \big(J_\nu(y)\big)_{ni}\ \Theta(y_0-x_0) \ . \end{aligned} \tag{5}$$