A1) Nephroide

$$\vec{r}(t) = R\begin{pmatrix} 3\cos\omega t - \cos(3\omega t) \\ 3\sin\omega t - \sin(3\omega t) \end{pmatrix}$$

a) $v_1(t)$, $|v(t)|$, $a(t)$, $|a(t)|$

$$v(t) = 3R\omega\begin{pmatrix} -\sin\omega t + \sin(3\omega t) \\ \cos\omega t - \cos(3\omega t) \end{pmatrix}$$

$$|v(t)| = \sqrt{9R^2\omega^2((\sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta) + (\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta))}$$

$$= \sqrt{9R^2\omega^2(2 - 2\underbrace{(\sin\alpha\sin\beta + \cos\alpha\cos\beta)}_{\cos(\alpha-\beta)})}$$

$$= 3R\omega\sqrt{2 - 2\cos(\alpha-\beta)} \qquad \cos(\alpha-\beta) = 1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 3R\omega\sqrt{2 - 2\cdot\left(1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)\right)} = 3R\omega\sqrt{2 - 2 + 4\sin^2\left(\frac{\alpha-\beta}{2}\right)}$$

$$= 6R\omega\sin\left(\frac{\omega t}{2}\right)$$

$$a(t) = 3R\omega^2\begin{pmatrix} -\cos\omega t + 3\cos(3\omega t) \\ -\sin\omega t + 3\sin(3\omega t) \end{pmatrix}$$

$$|a(t)| = \sqrt{9R^2\omega^4((\cos^2\alpha - 6\cos\alpha\cos\beta + 9\cos^2\beta) + (\sin^2\alpha - 6\sin\alpha\sin\beta + 9\sin^2\beta))}$$

$$= \sqrt{9R^2\omega^4(10 - 6(\cos\alpha\cos\beta + \sin\alpha\sin\beta))}$$

$$3R\omega^2\sqrt{10 - 6\cos(\alpha-\beta)} \qquad \cos(\alpha-\beta) = 1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$3R\omega^2\sqrt{10 - 6\left(1 - \sin^2\left(\frac{\alpha-\beta}{2}\right)\right)}$$

$$3R\omega^2\sqrt{4 + 6\sin^2\left(\frac{\alpha-\beta}{2}\right)}$$

b) $s(t)$ von $t_1 = 0$ bis $t_2 = \frac{2\pi}{\omega}$

$$\int_0^{\frac{2\pi}{\omega}} |v(t)|\,dt \Rightarrow \int_0^{\frac{2\pi}{\omega}} 6R\omega\sin\left(\frac{\omega t}{2}\right)dt = \left[-12R\cancel{\omega}^2\cos\left(\frac{\omega t}{2}\right)\right]_0^{\frac{2\pi}{\omega}}$$

$$= -12R\cancel{\omega}^2\cos\left(\frac{\cancel{\omega}2\pi}{2\cancel{\omega}}\right) - (-12R\omega^2\cos(0))$$

$$12R\cancel{\omega}^2 + 12R\cancel{\omega}^2$$

$$\underline{= 24R\cancel{\omega}^2}$$

$$s = |v(t)|\cdot t$$

$$t = \frac{s}{|v(t)|}$$

0

b) $F(r) = A(x^2+y^2+z^2)^{\frac{1}{2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad c_2(-\cos\omega t, \sin\omega t, 0) \quad 0 \le t \le \frac{\pi}{\omega}$

$$\Delta W = -\int_0^{\pi/\omega} (\cos^2\omega t + \sin^2\omega t)^{\frac{1}{2}} \begin{pmatrix} -\cos\omega t \\ \sin\omega t \end{pmatrix} \cdot \begin{pmatrix} \omega \sin\omega t \\ \omega \cos\omega t \end{pmatrix} dt$$

$$= -\int_0^{\pi/\omega} (\cos^2\omega t + \sin^2\omega t)^{\frac{1}{2}} (-\cos\omega t \cdot \omega \cdot \sin\omega t) + (\cos^2\omega t + \sin^2\omega t)^{\frac{1}{2}} (\sin\omega t \cdot \omega \cos\omega t) \, dt$$

$$= 0$$

c) $\vec{F}(\vec{r}) = (-Ay, Ax, 0) \quad c_1(x, 0, 0) \quad c_2(-\cos\omega t, \sin\omega t, 0)$

$$\Delta W_1 = -\int_{-1}^{1} \begin{pmatrix} 0 \\ Ax \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} dx$$ $\Rightarrow 0$, da Skalar $= 0$ ist.

$$\Delta W_2 = -\int_0^{\pi/\omega} \begin{pmatrix} -\sin\omega t \\ -\cos\omega t \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \omega \sin\omega t \\ \omega \cos\omega t \\ 0 \end{pmatrix} dt$$

$$-A\int_0^{\pi/\omega} (-\omega \sin^2\omega t - \omega\cos^2\omega t) \, dt = -A\int_0^{\pi/\omega} -\omega \underbrace{(\sin^2\omega t + \cos^2\omega t)}_{1} dt$$

$$= -A\int_0^{\pi/\omega} -\omega \, dt = -A\left[-\omega t\right]_0^{\pi/\omega} = -A\left(-\omega\frac{\pi}{\omega}\right) - 0$$

$$= A\pi$$

c) ✓

d) $\vec{e_t}$, $\vec{e_N}(t)$, k, ρ, $\vec{a}(t)$ neu ausdrücken.

$$\vec{t} = \frac{d\vec{r}(s)}{ds} = \dot{r}(s) = R\begin{pmatrix} 3\cos\left(\frac{\omega s}{|v(t)|}\right) - \cos\left(\frac{3\omega s}{|v(t)|}\right) \\ 3\sin\left(\frac{\omega s}{|v(t)|}\right) - \sin\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}$$

$$\vec{t} = \frac{3R\omega}{|v(t)|}\begin{pmatrix} -\sin\left(\frac{\omega s}{|v(t)|}\right) + \sin\left(\frac{3\omega s}{|v(t)|}\right) \\ \cos\left(\frac{\omega s}{|v(t)|}\right) - \cos\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}$$

$$\vec{e_t} = \frac{\vec{t}}{|\vec{t}|}$$

$$|\vec{t}| = \frac{3R\omega}{|v(t)|}\sqrt{\sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta + \cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta}$$

$$= \frac{3R\omega}{|v(t)|}\sqrt{2 - 2(\underbrace{\sin\alpha\sin\beta + \cos\alpha\cos\beta}_{\cos(\alpha-\beta)})}$$

$$\cos(\alpha-\beta) = 1 - 2\sin^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= \frac{3R\omega}{|v(t)|}\sqrt{2-2\left(1-2\sin^2\left(\frac{\alpha-\beta}{2}\right)\right)} = \frac{3R\omega}{|v(t)|}\sqrt{4\sin^2\left(\frac{\alpha-\beta}{2}\right)}$$

$$\vec{e_t} = \frac{\cancel{\frac{3R\omega}{|v(t)|}}\begin{pmatrix} -\sin\left(\frac{\omega s}{|v(t)|}\right) + \sin\left(\frac{3\omega s}{|v(t)|}\right) \\ \cos\left(\frac{\omega s}{|v(t)|}\right) - \cos\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}}{\cancel{\frac{3R\omega}{|v(t)|}}\, 2\sin\left(\frac{\omega t}{2}\right)}$$

$$\vec{e_N} = \frac{\frac{d\vec{e_t}(s)}{ds}}{\left|\frac{d\vec{e_t}(s)}{ds}\right|} = \frac{3R\omega^2}{|v(t)|^2}\begin{pmatrix} -\cos\left(\frac{\omega s}{|v(t)|}\right) + 3\cos\left(\frac{3\omega s}{|v(t)|}\right) \\ -\sin\left(\frac{\omega s}{|v(t)|}\right) + 3\sin\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}$$

$$\left|\frac{d\vec{e_t}(s)}{ds}\right| = \frac{3R\omega^2}{|v(t)|^2}\sqrt{\cos^2\alpha - 6\cos\alpha\cos\beta + 9\cos^2\beta + \sin^2\alpha - 6\sin\alpha\sin\beta + 9s}$$

$$= \frac{3R\omega^2}{|v(t)|^2}\sqrt{10 - 6(\cos(\alpha-\beta))} = \frac{3R\omega^2}{|v(t)|^2}\sqrt{4 + 6\sin^2\left(\frac{\alpha-\beta}{2}\right)}$$

$$\vec{e_N} = \frac{\begin{pmatrix} -\cos\left(\frac{\omega s}{|v(t)|}\right) + 3\cos\left(\frac{3\omega s}{|v(t)|}\right) \\ -\sin\left(\frac{\omega s}{|v(t)|}\right) + 3\sin\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}}{\sqrt{4 + 6\sin^2\left(\frac{\alpha-\beta}{2}\right)}}$$

$$K = \frac{3Rw^2}{|v(t)|^2}\sqrt{4+6\sin^2\left(\frac{\alpha-\beta}{2}\right)}$$

$$\rho = \frac{1}{K} \Rightarrow \frac{|v(t)|^2}{3Rw^2\sqrt{4+6\sin^2\left(\frac{\alpha-\beta}{2}\right)}}$$

$$a(t) = \underbrace{\frac{d|v(t)|}{dt}}_{0} e_T + \frac{|v(t)|^2}{\rho} e_T$$

$$\Rightarrow \frac{\cancel{|v(t)|^2}\, 3R\omega^2\sqrt{4+6\sin^2\left(\frac{\alpha-\beta}{2}\right)}}{\cancel{|v(t)|^2}} \cdot \begin{pmatrix} -\sin\left(\frac{\omega s}{|v(t)|}\right) + 3\sin\left(\frac{3\omega s}{|v(t)|}\right) \\ \cos\left(\frac{\omega s}{|v(t)|}\right) - 3\cos\left(\frac{3\omega s}{|v(t)|}\right) \end{pmatrix}$$

$2\sin\left(\frac{\omega t}{2}\right)$ → geht leider nicht ganz auf.