

$$\tau = \frac{1}{c} \int_{s_1}^{s_2} \sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu} ds$$

$$\delta\tau = 0$$

$$\delta\tau = \frac{d}{d\epsilon} \tau(x^\kappa + \epsilon h^\kappa)|_{\epsilon=0}$$

$$\begin{aligned} &= \frac{1}{c} \int_{s_1}^{s_2} \frac{d}{d\epsilon} \sqrt{g_{\mu\nu}(x^\lambda + \epsilon h^\lambda)(\dot{x}^\mu + \epsilon \dot{h}^\mu)(\dot{x}^\nu + \epsilon \dot{h}^\nu)} ds|_{\epsilon=0} = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} \frac{\partial_\lambda g_{\mu\nu}(x^\lambda + \epsilon h^\lambda) h^\lambda (\dot{x}^\mu + \epsilon \dot{h}^\mu)(\dot{x}^\nu + \epsilon \dot{h}^\nu) + g_{\mu\nu}(x^\lambda + \epsilon h^\lambda) \dot{h}^\mu (\dot{x}^\nu + \epsilon \dot{h}^\nu) + g_{\mu\nu}(x^\lambda + \epsilon h^\lambda) \dot{h}^\nu (\dot{x}^\mu + \epsilon \dot{h}^\mu)}{\sqrt{g_{\mu\nu}(x^\lambda + \epsilon h^\lambda)(\dot{x}^\mu + \epsilon \dot{h}^\mu)(\dot{x}^\nu + \epsilon \dot{h}^\nu)}} ds|_{\epsilon=0} = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) h^\lambda \dot{x}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \dot{h}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \dot{h}^\nu \dot{x}^\mu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds \end{aligned}$$

Umsortieren ergibt fuer $\dot{h}$ mit einheitlichem Index: $\dot{h}^\mu \dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda)) = g_{\mu\nu}(x^\lambda) \dot{h}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \dot{h}^\nu \dot{x}^\mu$

$$\begin{aligned} \delta\tau &= \frac{1}{2c} \int_{s_1}^{s_2} h^\lambda \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds + \frac{1}{2c} \int_{s_1}^{s_2} \dot{h}^\mu \frac{\dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda))}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} h^\lambda \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{d}{ds} \frac{\dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda))}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} h^\lambda \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds \\ &\quad - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{(\ddot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda)) + \dot{x}^\nu (\partial_\lambda g_{\mu\nu}(x^\lambda) + \partial_\lambda g_{\nu\mu}(x^\lambda))) \sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}}{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu} ds \\ &\quad - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda)) \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \ddot{x}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \ddot{x}^\nu \dot{x}^\mu}{2\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}}}{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu} ds = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} h^\lambda \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds \\ &\quad - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda)) + \dot{x}^\nu (\partial_\lambda g_{\mu\nu}(x^\lambda) + \partial_\lambda g_{\nu\mu}(x^\lambda))}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds \\ &\quad - \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \dot{x}^\nu (g_{\mu\nu}(x^\lambda) + g_{\nu\mu}(x^\lambda)) \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\lambda \dot{x}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \ddot{x}^\mu \dot{x}^\nu + g_{\mu\nu}(x^\lambda) \ddot{x}^\nu \dot{x}^\mu}{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu \sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds = \\ &= \frac{1}{2c} \int_{s_1}^{s_2} h^\lambda \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\ddot{x}^\nu g_{\mu\nu}(x^\lambda)}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\ddot{x}^\nu g_{\nu\mu}(x^\lambda)}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \\ &\quad \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\dot{x}^\nu \partial_\lambda g_{\mu\nu}(x^\lambda)}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{2c} \int_{s_1}^{s_2} h^\mu \frac{\dot{x}^\nu \partial_\lambda g_{\nu\mu}(x^\lambda)}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\lambda \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{g_{\mu\nu}(x^\lambda) \ddot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \\ &\quad \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{g_{\mu\nu}(x^\lambda) \ddot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{\partial_\lambda g_{\mu\nu}(x^\lambda) \dot{x}^\lambda \dot{x}^\nu \frac{g_{\nu\mu}(x^\lambda)}{g_{\mu\nu}(x^\lambda)}}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{g_{\nu\mu}(x^\lambda) \ddot{x}^\mu \dot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds - \\ &\quad \frac{1}{4c} \int_{s_1}^{s_2} h^\mu \frac{g_{\nu\mu}(x^\lambda) \ddot{x}^\nu}{\sqrt{g_{\mu\nu}(x^\lambda) \dot{x}^\mu \dot{x}^\nu}} ds = \end{aligned}$$

